Magnetic Circuits

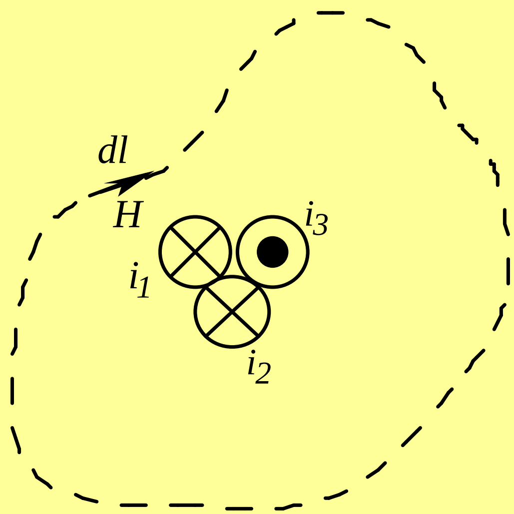
- Ampere's Law
- Ampere-Turns
- Magnetic-Field Intensity, Flux Density
- Reluctance
- Inductance
- Faraday's Law
- Magnetizing Flux and the Leakage Flux

Magnetic Field

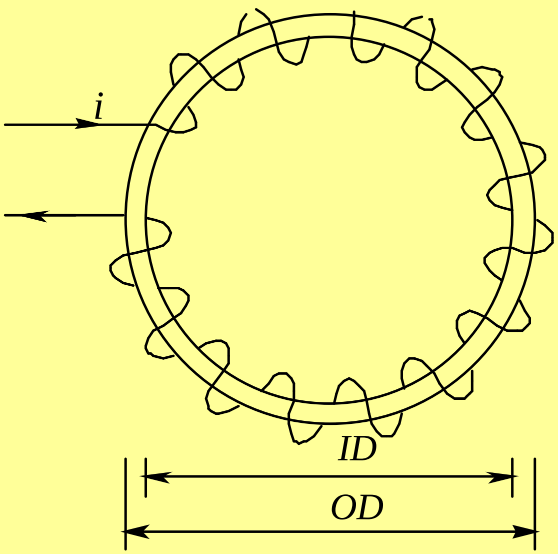
- Magnetic field, *H*, produced by current carrying conductor

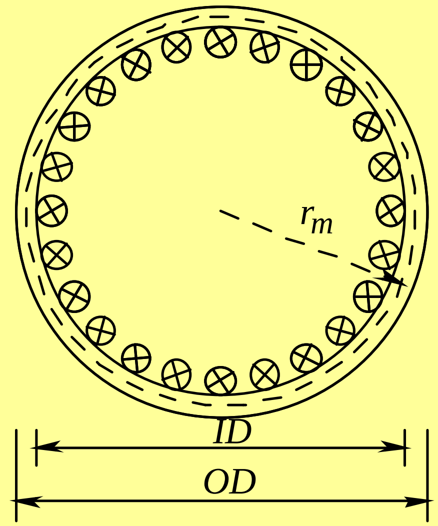
- Ampere's Law

$$\oint_{closed\ path} H\, d\ell = \sum i$$



H in a Toroid





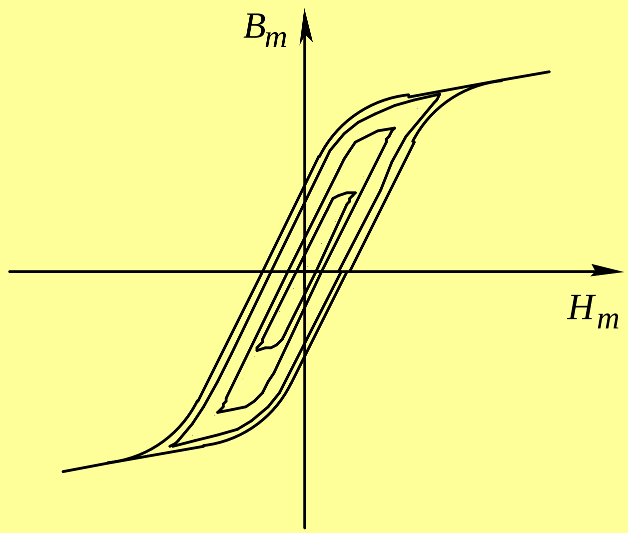
$$\text{Mean radius, } r_m = \frac{1}{2}\left(\frac{ID + OD}{2}\right)$$

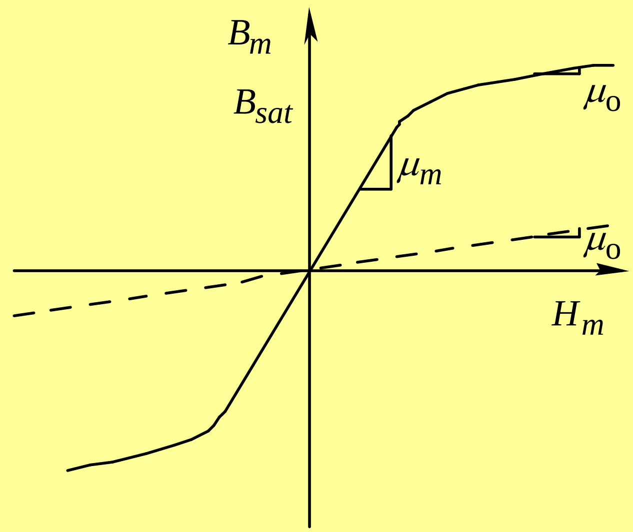
$$l_m = 2\pi r_m$$

$$\text{Ampere's Law } \Rightarrow H_m = \frac{Ni}{2\pi r_m} = \frac{Ni}{l_m}$$

Flux Density *B*

- ☐ Units: $Weber/meter^2\ [Wb/m^2]$ or $Tesla[T]$
- ☐ In air $B=\mu_o H$, $\mu_o = 4\pi\times10^{-7}\left[\frac{henries}{m}\right]$
- ☐ Ferro-magnetic materials





- ◆ Linear approximation

$$B_m = \mu_m H_m$$

- ◆ B_{sat} ~ *1.6* - 1.8 Tesla
- ◆ In saturation

μ_m approaches μ_o

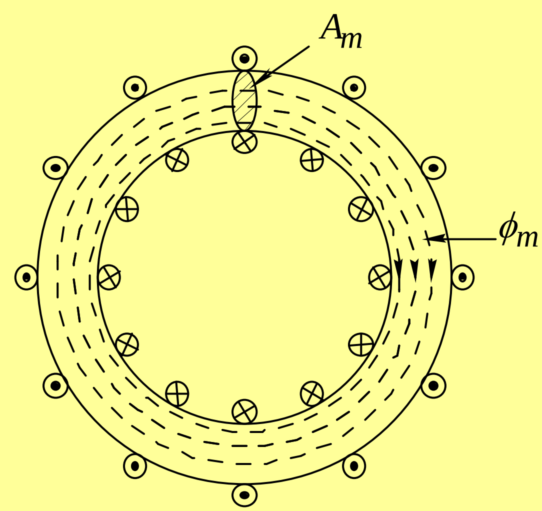
Flux, Flux Linkage, and MMF

- Flux ϕ_m $[Wb]$
  [assuming uniform flux density]

$$\phi_m = B_m A_m$$

$$B_m = \mu_m H_m \text{ and } H_m = \frac{Ni}{\ell_m}$$

$$\therefore \ \phi_m = A_m \left(\mu_m \frac{Ni}{\ell_m} \right) = \frac{Ni}{\left(\frac{\ell_m}{\mu_m A_m} \right)} = \frac{F}{\Re_m}$$



- Reluctance

$$\Re_m = \frac{\ell_m}{\mu_m A_m}$$

- Flux Linkage

$$\lambda_m = N \phi_m$$

- MMF

$$F = Ni$$

Magnetic Structures with Air Gaps

$$H_m \ell_m + H_g \ell_g = N i$$

$$B_m = \mu_m H_m \ , \ \ B_g = \mu_o H_g$$

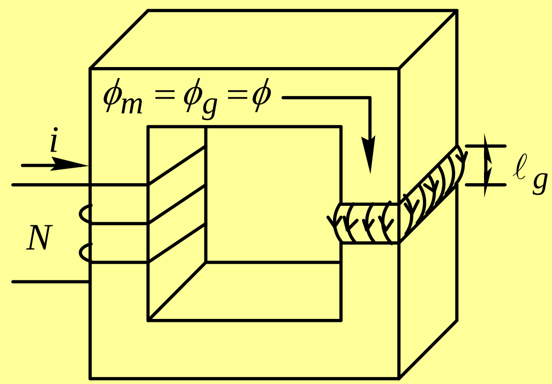
$$\frac{B_m}{\mu_m}\ell_m + \frac{B_g}{\mu_o}\ell_g = N i$$

$$\phi = A_m B_m = A_g B_g$$

$$B_m = \frac{\phi}{A_m} \qquad B_g = \frac{\phi}{A_g}$$

$$\phi_m \Big(\underbrace{\frac{\ell_m}{A_m \mu_m}}_{\Re_m} + \underbrace{\frac{\ell_g}{A_g \mu_o}}_{\Re_g} \Big) = N i$$

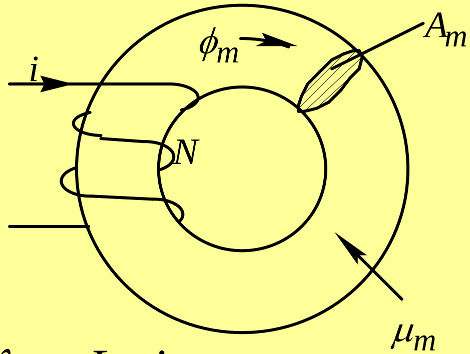
$$\phi_m = \frac{F}{\Re}$$



To account for fringing $A_g = (w + \ell_g)(d + \ell_g)$

$$\Re = \Re_m + \Re_g$$

Inductance



$$i \xrightarrow{\times\left(\frac{N}{l_m}\right)} H_m \xrightarrow{\times(\mu_m)} B_m \xrightarrow{\times(A_m)} \phi_m \xrightarrow{\times(N)} \lambda_m$$

$$i \xrightarrow{\times\left(L_m = \frac{N^2}{\frac{l_m}{\mu_m A_m}}\right)} \lambda_m$$

$$\lambda_m = L_m i$$

$$L_m = \frac{\lambda_m}{i} = \left(\frac{N}{l_m}\right) \mu_m A_m \; N = \frac{N^2}{\left(\frac{l_m}{\mu_m A_m}\right)} = \frac{N^2}{\Re}$$

- For linear magnetic conditions inductance depends only on magnetic circuit

☐ Energy stored in magnetic circuits

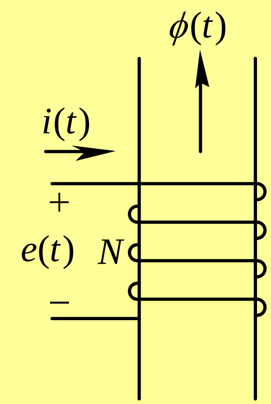
$$W = \frac{1}{2} L i^2 = \frac{1}{2\mu_m} B_m^2 \underbrace{A_m l_m}_{\text{volume}}$$

☐ Energy density

$$w = \frac{W}{\text{volume}} = \frac{1}{2\mu_m} B_m^2$$

Faraday's Law - Induced Voltage

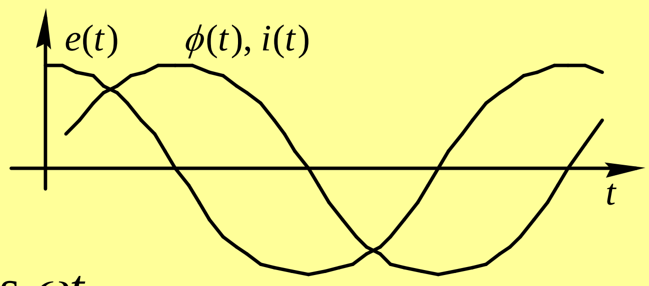
- ☐ Induced voltage $e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}$
- ☐ Current direction is into positive polarity voltage to define the flux direction
- ☐ Lenz's law: Polarity of induced voltage

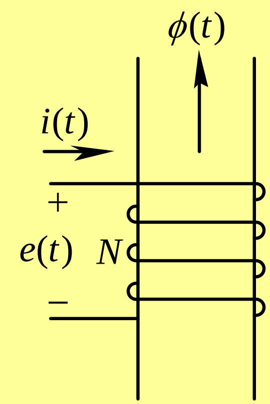


Coil in Sinusoidal Steady-State

- Induced voltage under sinusoidal steady-state

  Given





$$\phi(t) = \hat{\phi} \sin \omega t$$

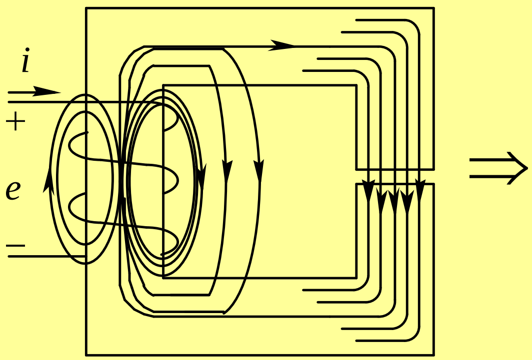
$$e(t) = N\frac{d\phi}{dt} = N\,\hat{\phi}\,\omega \cos \omega t$$

- Relating $e(t)$, $\phi(t)$, and $i(t)$

$$L = \frac{\lambda}{i} = \frac{N\phi}{i}$$

$$\left.\begin{aligned} \Rightarrow\ & i(t) = \frac{N}{L}\phi(t) \\ \&\ \ & e(t) = N\frac{d\phi(t)}{dt} \end{aligned}\right\} \Rightarrow e(t) = L\,\frac{di(t)}{dt}$$

Leakage and Magnetizing Inductances



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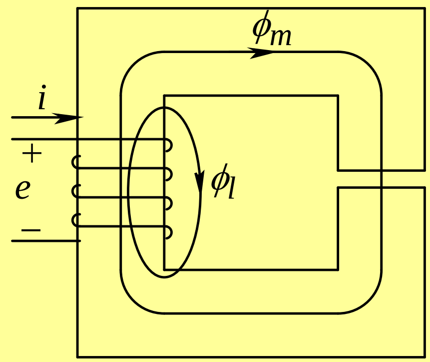
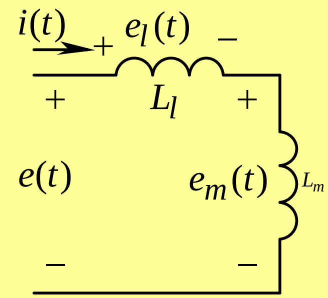


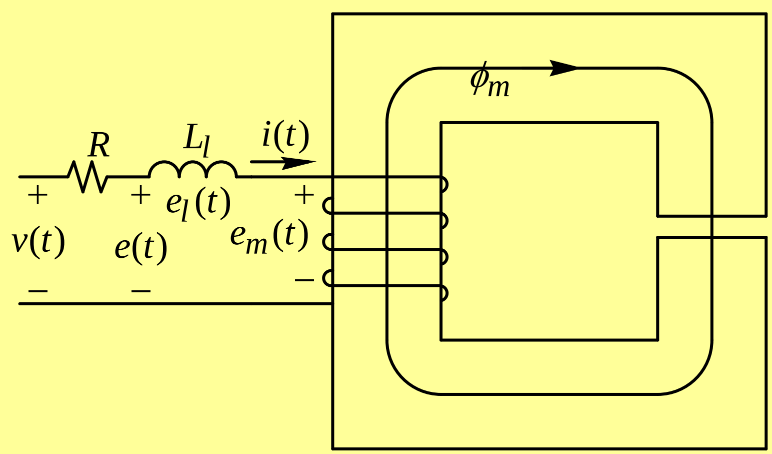


$$\phi = \phi_m + \phi_\ell \qquad \lambda = N\phi = \underbrace{N\,\phi_m}_{\lambda_m} + \underbrace{N\,\phi_\ell}_{\lambda_\ell}$$

$$\frac{\lambda}{i} = \frac{\lambda_m}{i} + \frac{\lambda_\ell}{i} \qquad \Rightarrow L_{self} = L_m + L_\ell$$

$$\lambda = L_{self}\, i = L_m i + L_\ell i$$

$$e = \underbrace{L_m \frac{di}{dt}}_{e_m} + \underbrace{L_\ell \frac{di}{dt}}_{e_\ell} = e_m + L_\ell \frac{di}{dt}$$



Summary

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- Ampere's Law
- Ampere-Turns
- Magnetic-Field Intensity, Flux Density
- Reluctance
- Inductance
- Faraday's Law
- Magnetizing Flux and the Leakage Flux