Week 12:
Output Stages, Frequency Response

(2 hours only)

Topics to cover …

- Output Stages
- Amplifier Frequency Response

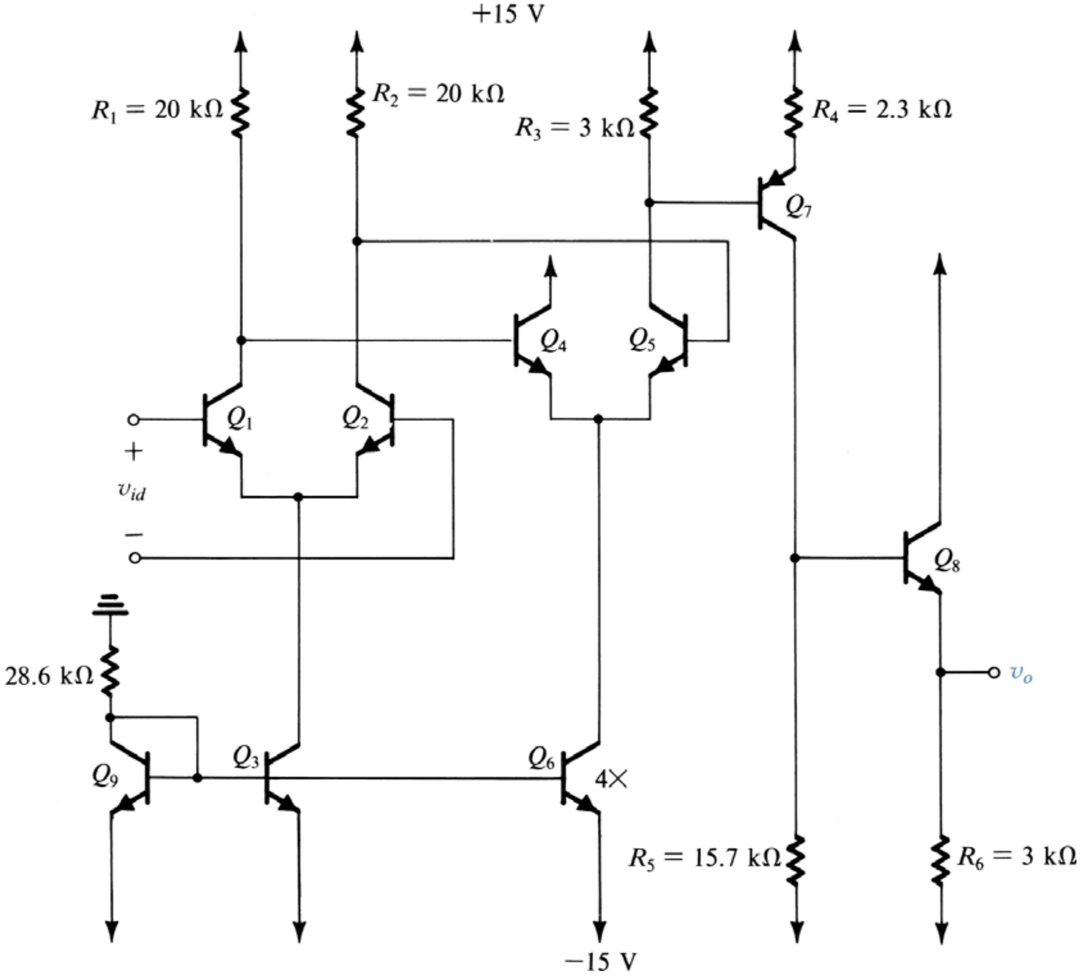
Reading Assignment:
Chap 15.3, 16.1 of Jaeger and Blalock or
Chap 14.1 – 14.4 of Sedra & Smith

Multistage Amplifiers

Practical amplifiers usually consist of a number of stages connected in cascade.

- The first (input) stage is usually required to provide
  - a high input resistance
  - a high common-mode rejection for a differential amplifier
- Middle stages are to provide
  - majority of voltage gain
  - conversion of the signal from differential mode to single-end mode
  - shifting of the dc level of the signal
- The last (output) stage is to provide
  - a low output resistance in order to
    - avoid loss of gain and
    - provide the current required by the load (power amplifiers)

Example

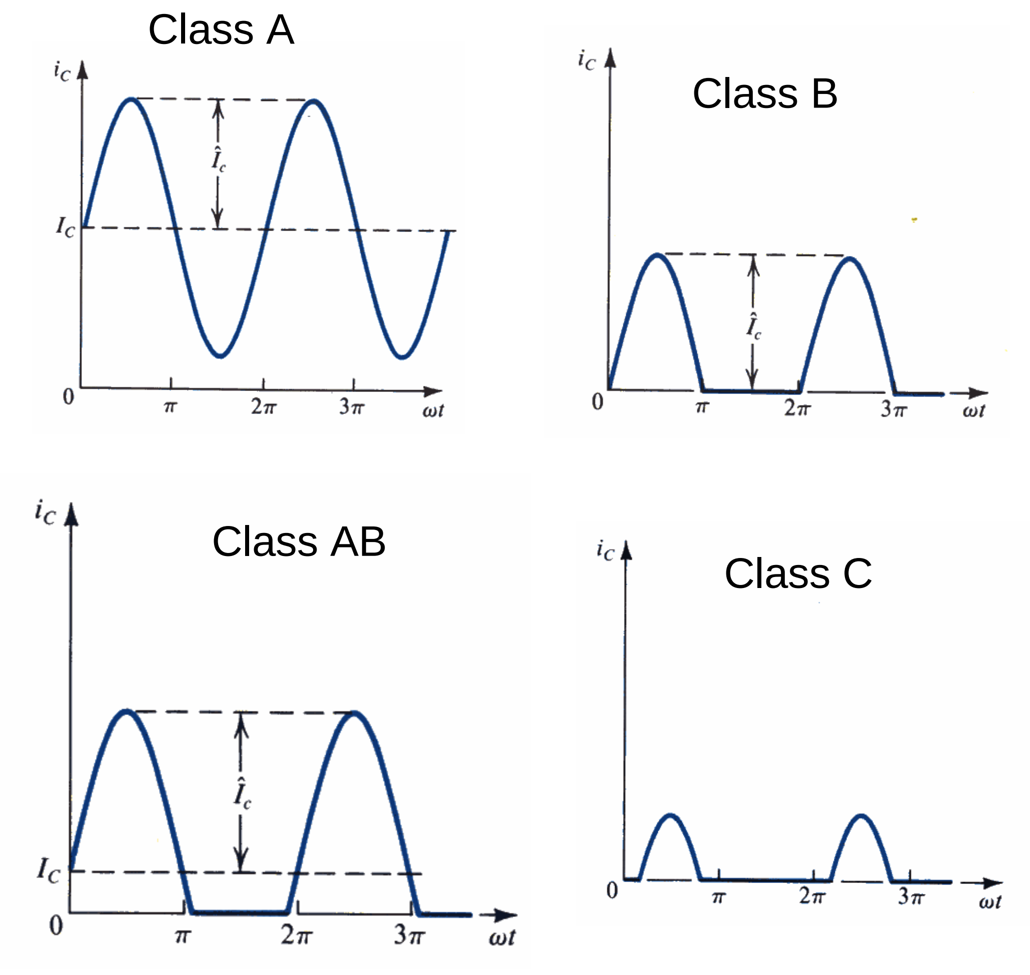


- The input stage (Q_1, Q_2) is differential-in and differential-out
  - biased by current source Q_3
- (Q_4, Q_5) is a differential-in and single-ended-out stage
  - biased by current source Q_6
- Q_7 provides
  - additional gain
  - shifting the dc level of the signal
- The output stage Q_8 is an emitter follower

Output Stages

- Function of an output stage is
  - To provide a low output resistance so that it can deliver the output signal to the load without loss of gain
- Requirements of an output stage:
  - Large input signal range
    - b/c it is the final stage of the amplifier, and usually deals with relatively large signals.
    - Small-signal approximations and models either are not applicable or must be used with care.
  - Low distortion
  - High power efficiency

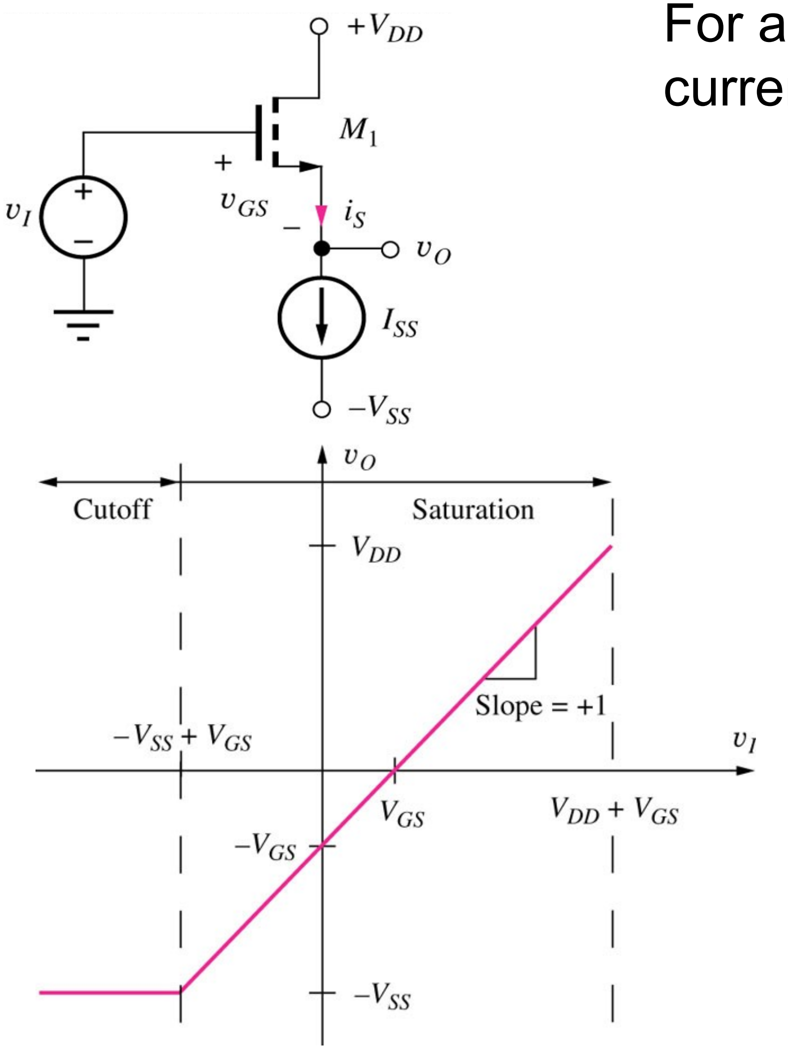
Classification of Output Stages



Collector or Drain current waveforms of different output stages

- Class A: the transistor conducts for the entire cycle of the input signal
- Class B: the transistor conducts for only half the cycle
- Class AB: conduction cycle is greater than 180^o and less than 360^o
  - Used for opamp output stage and audio power amplifiers
- Class C: conduction cycle is less than 180^o
  - Used for radio-frequency (RF) power amplifications (mobile phones, radio and TV)

Class-A Amplifier: Source/Emitter Follower



For a source follower biased by an ideal current source, v_{GS} is fixed and

$$v_O = v_I - V_{GS1} = v_I - \left(V_{TN} + \sqrt{\frac{2I_{SS}}{K_n}} \right)$$

Input range:

$$-V_{SS} + V_{GS} \le v_I \le V_{DD} + V_{TN}$$

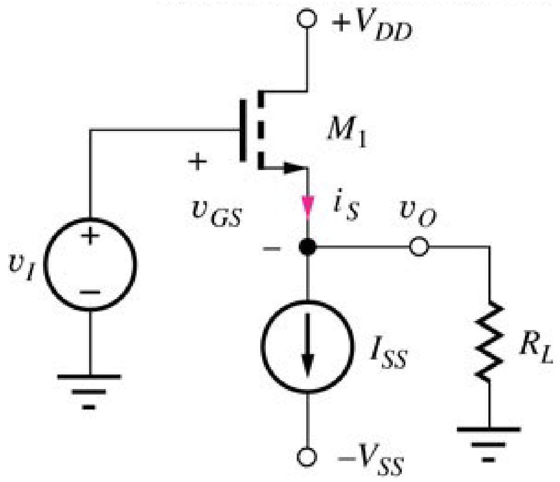
Output range:

$$-V_{SS} \le v_O \le V_{DD}$$

The largest output voltage is

$$v_O \cong V_{DD} \sin \omega t \qquad \text{(if } V_{SS}=V_{DD}\text{)}$$

Source Emitter with Load



To maintain class A operation,
$i_s > 0$ at all times:

$$\therefore \quad i_S = I_{SS} + \frac{v_O}{R_L} \geq 0$$

$$v_o \geq -I_{SS} R_L$$

For largest output amplitude: $v_O \cong V_{DD} \sin \omega t$

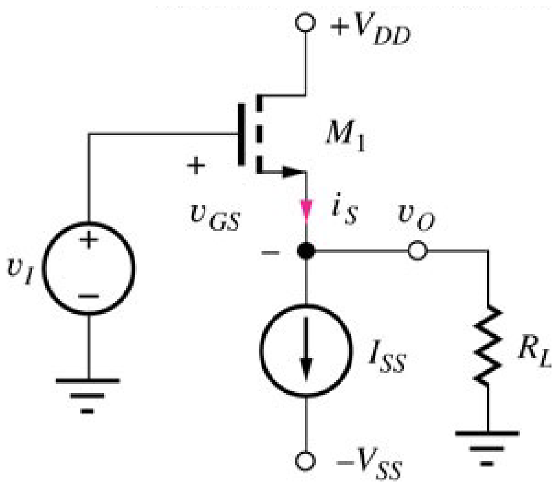
We have: $V_{DD} \sin \omega t \geq -I_{SS} R_L \quad \text{for all } t$

The lowest value for the LHS occurs when sin ωt = -1,

$$\therefore V_{DD} \leq I_{SS} R_L$$

$$\therefore I_{SS} \geq \frac{V_{DD}}{R_L}$$

Power Efficiency



The largest output voltage is

$$v_O \cong V_{DD} \sin \omega t$$

Average power supplied to the source follower:

$$P_{av} = \frac{1}{T}\int_0^T \left[I_{SS}\left(V_{DD} + V_{SS}\right) + \left(\frac{V_{DD}\sin \omega t}{R_L}\right) V_{DD} \right] dt$$

$$= I_{SS}\left(V_{DD} + V_{SS}\right) = 2I_{SS}V_{DD}$$

(if V_{SS}=V_{DD})

Average power delivered to the load:

$$P_{ac} = \frac{\left(\frac{V_{DD}}{\sqrt{2}}\right)^2}{R_L} = \frac{V_{DD}^2}{2R_L}$$

Efficiency of amplifier is:

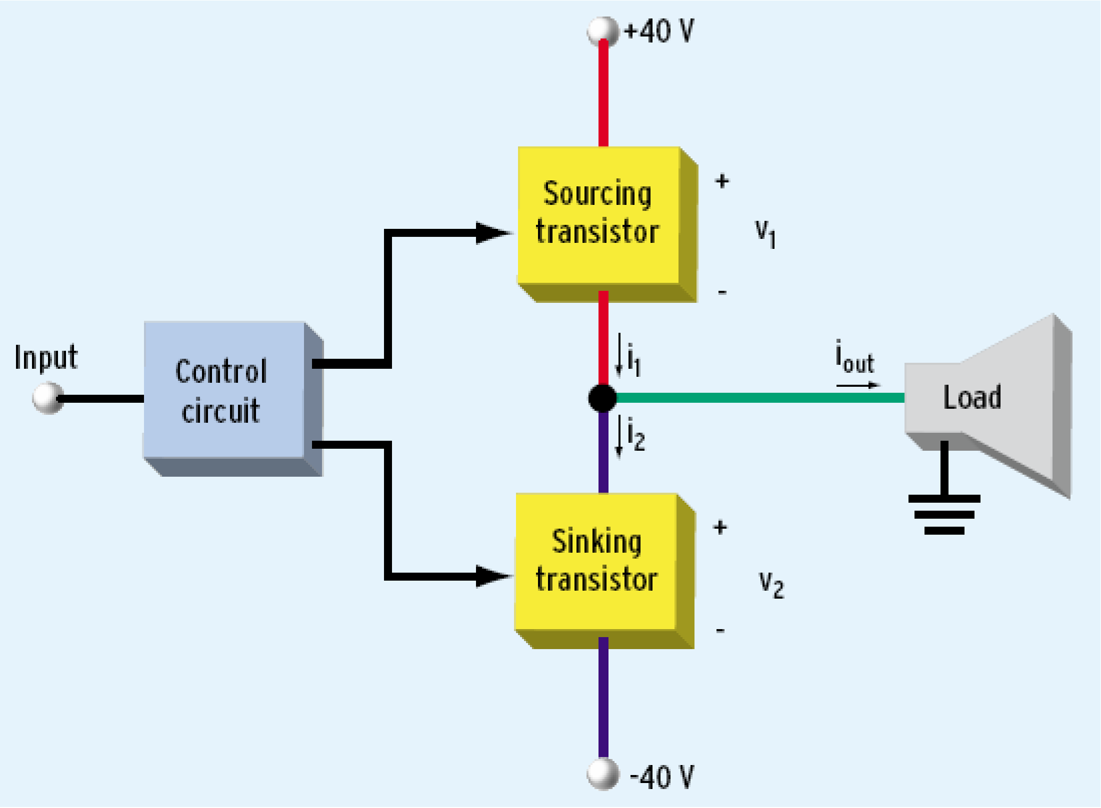
$$\zeta = \frac{P_{\mathrm{ac}}}{P_{\mathrm{av}}} = \frac{V_{DD}^2/(2R_L)}{2I_{SS}V_{DD}}$$

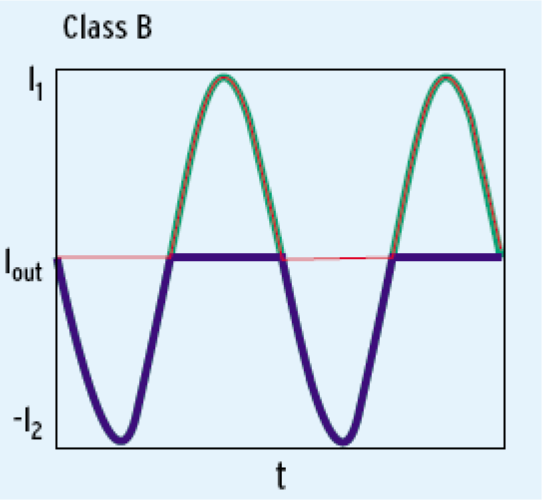
$$\leq \frac{V_{DD}^2/(2R_L)}{2\left(V_{DD}/R_L\right)V_{DD}} = 25\%$$

$$I_{SS} \geq \frac{V_{DD}}{R_L}$$

- Low efficiency

Push-Pull Operation: Class B

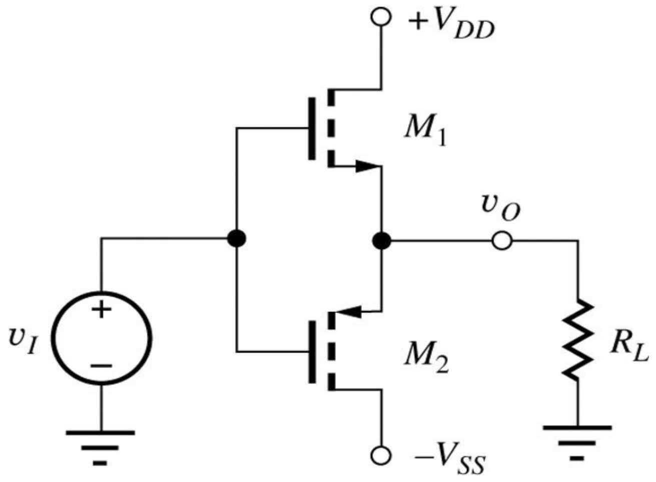




When a push-pull amplifier is operated in Class B, all of the output current comes either from the current-sourcing transistor or from the current-sinking device but never from both at the same time.

Source: B. Putzeys, “Digital Audio’s final frontier”, IEEE Spectrum, Mar 2003.

Class-B Amplifier



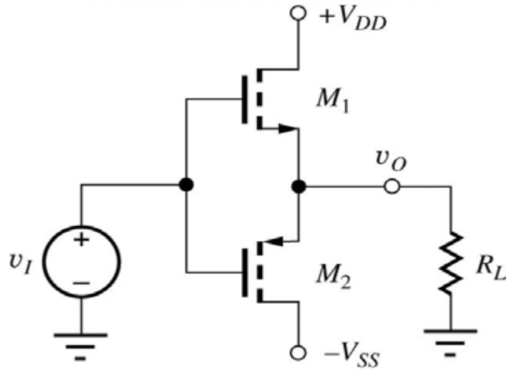
- A complementary pair of source followers biased at zero source current
- When

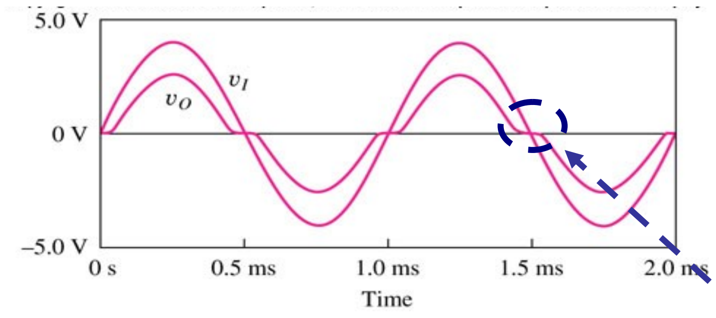
$$V_{TP} \le v_I \le V_{TN}$$

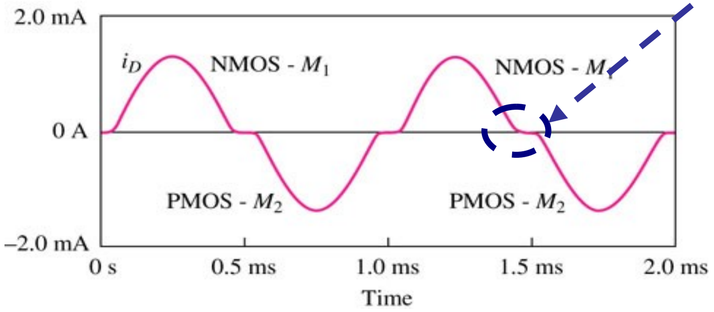
neither transistor conducts

- No quiescent (DC) current consumption!

Class-B Amplifier



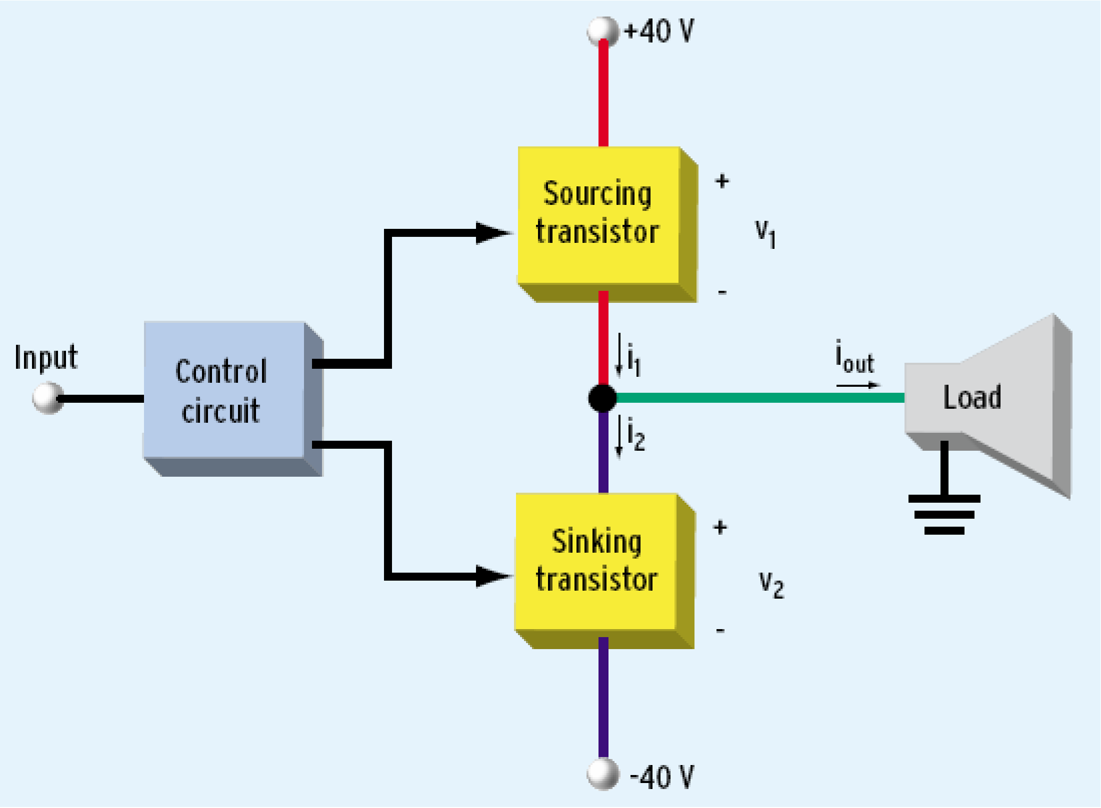


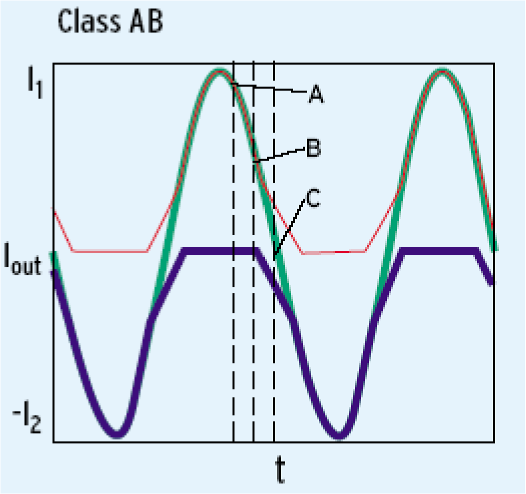


- When $V_I > V_{TN}$,
  - M_1 turns on and acts as an source follower, $v_o \approx v_I - V_{TN}$
  - M_2 off
- When $V_I < V_{TP}$,
  - M_2 turns on and acts as an source follower, $v_o \approx v_I – V_{TP}$
  - M_1 off
- Power efficiency is high, can be up to about 80%

- Disadv.: Output waveform suffers from a dead-zone → Large distortion

Class AB

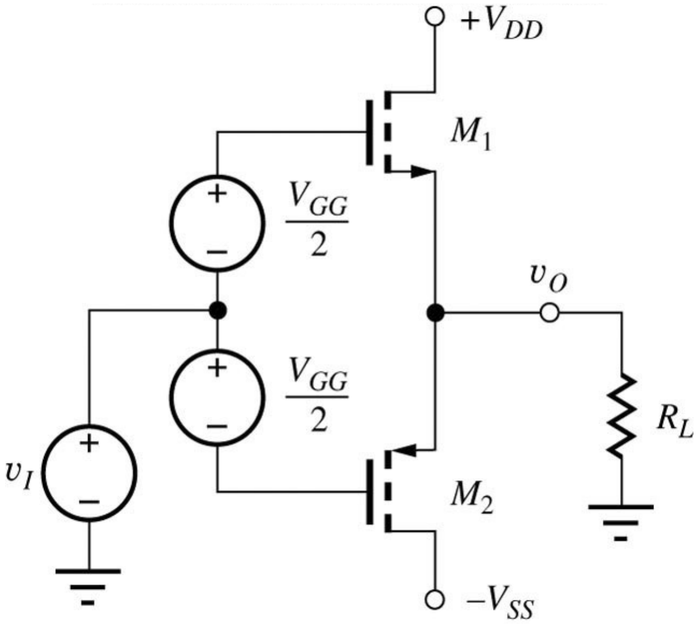




Class AB exhibits less distortion by allowing the transistors to work together when the output signal is near zero, in what is called the crossover region.

Source: B. Putzeys, “Digital Audio’s final frontier”, IEEE Spectrum, Mar 2003.

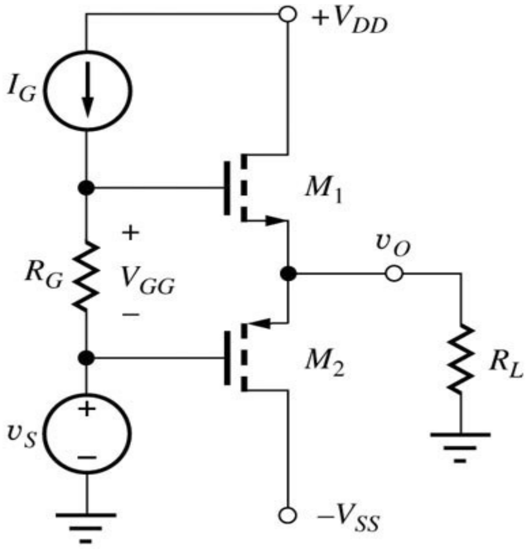
Class-AB Amplifiers



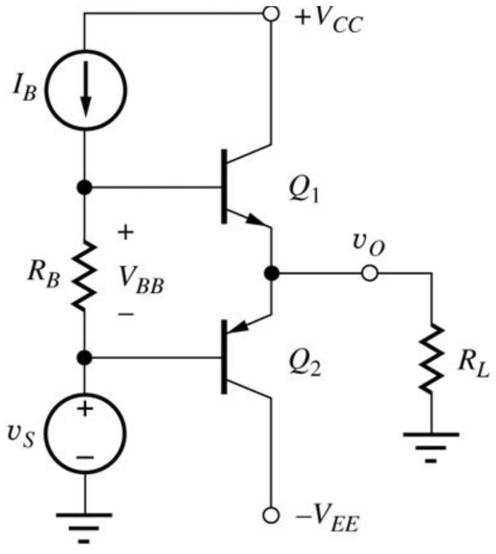
- Remove dead zone by biasing transistors into conduction but at a low quiescent current level
  - Distortion less than Class-B but worse than Class-A amplifier
- For each transistor, $180^0 < \text{conduction angle} < 360^0$ → Class AB amplifier
- Power efficiency lower than Class-B but higher than Class-A amplifier

Class-AB Amplifiers

Biasing examples:



(a)



(b)

DC currents:

$$I_D = \frac{K_n}{2}\left(\frac{V_{GG}}{2} - V_{TN}\right)^2$$

$$I_C = I_S \exp\left(\frac{I_B R_B}{2V_T}\right)$$

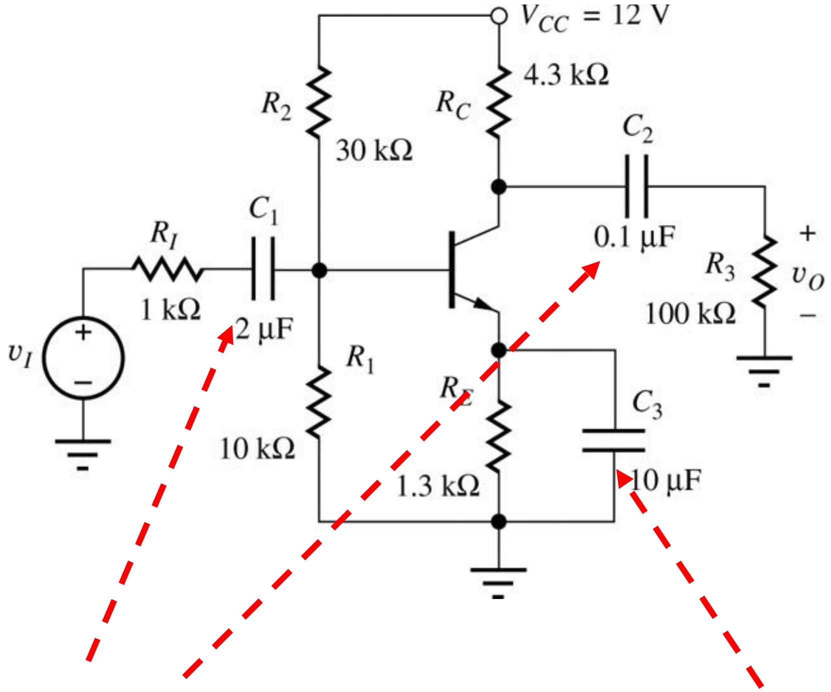
Topics to cover …

- Output Stages

- Amplifier Frequency Response

Frequency Response of Amplifiers

A typical amplifier:

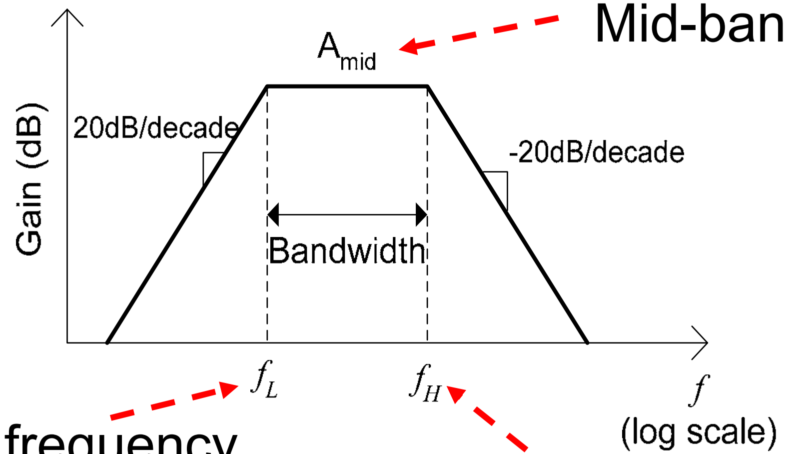


Block DC and low frequency signals

By-pass high frequency currents

Amplifier’s gain is frequency dependent!

Typical Amplifier Transfer Function



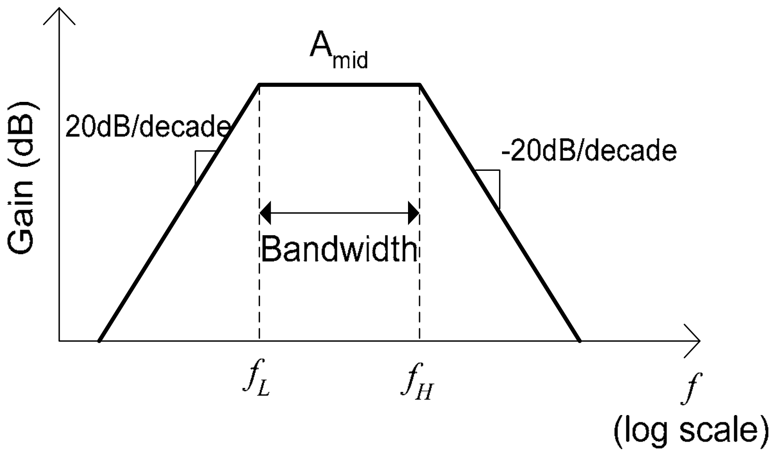
Mid-band gain

Lower cut-off frequency

Upper cut-off frequency

- In low frequency side, drop in gain is caused by coupling and bypass capacitors
- In high frequency side, drop in gain is caused by transistor’s parasitic capacitors
  - More on this topic later
- In the mid-band range, no capacitors are in effect:
  - Coupling and bypass capacitors are short circuits
  - Transistor parasitic capacitors are open circuits

Estimate f_L: Short-Circuit Time Constant Method



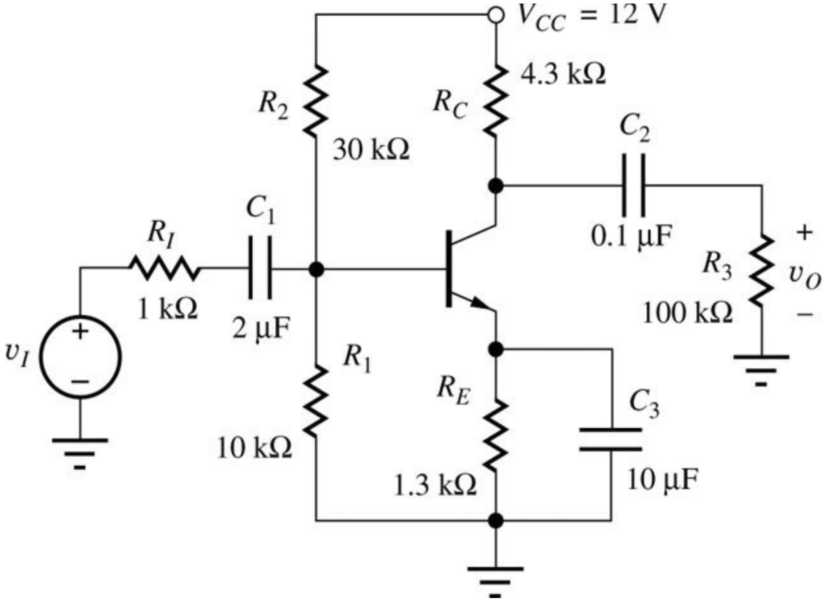
- Lower cutoff frequency for a network with *n* coupling and bypass capacitors can be estimated by:

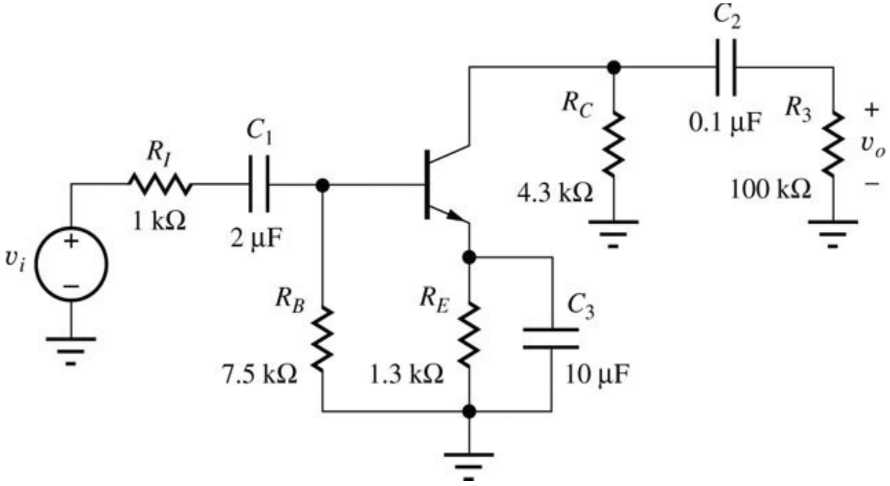
$$\boxed{\begin{aligned} \omega_L &\cong \sum_{i=1}^{n} \frac{1}{R_{iS} C_i} \\ f_L &= \omega_L / 2\pi \end{aligned}}$$

R_{iS} = resistance at terminals of i^{th} capacitor C_i with all other capacitors replaced by short circuits.

Product $R_{iS} C_i$ is "short-circuit time constant" associated with C_i.

SCTC: Example





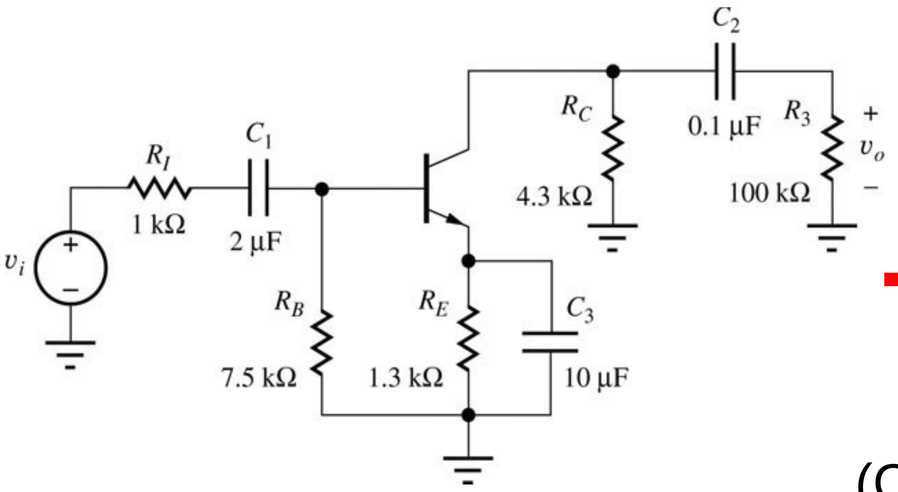
AC equivalent with **finite** coupling capacitances

β= 100 and V_A=75V
Q-point: (1.73mA, 2.32V)

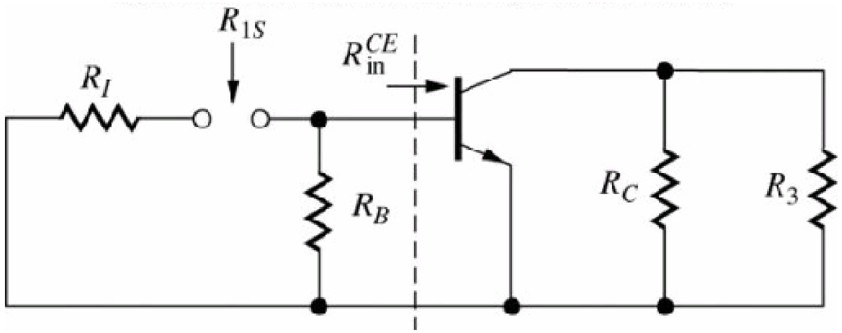
BJT small signal parameters:

$$r_\pi = \frac{V_T}{I_B} = \frac{25mV}{1.73mA/100} = 1.45k\Omega \qquad r_o = \frac{V_A + V_{CE}}{I_C} = \frac{75V + 2.32V}{1.73mA} = 44.7k\Omega$$

Time Constant Associated with C_1



To find the time constant associated with C_1:



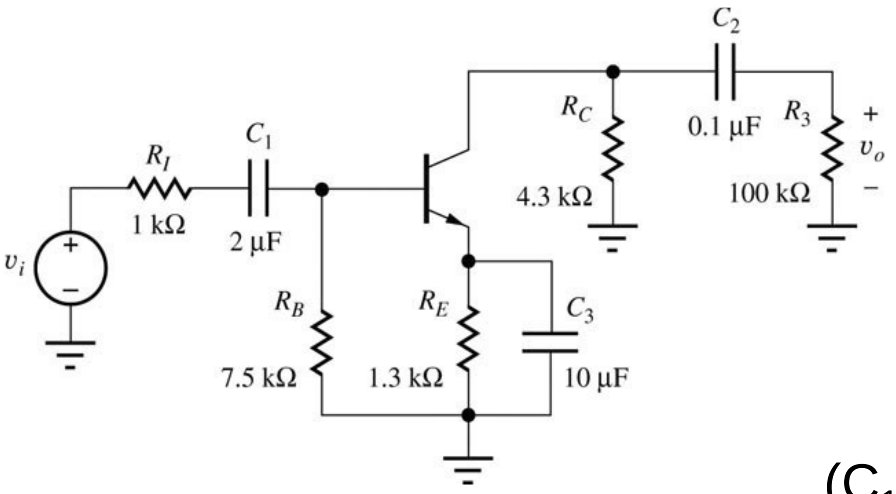
(C_2 and C_3 are short-circuited and set $v_i = 0$)

$$R_{1S} = R_I + (R_B \| R_{in}^{CE}) = R_I + (R_B \| r_\pi)$$

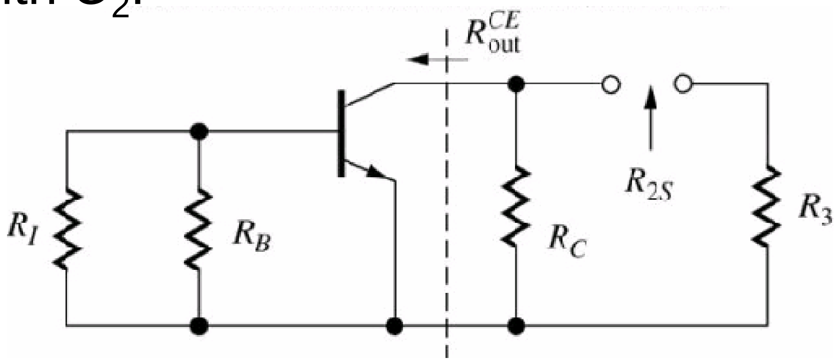
$$R_{1s} = 1000\Omega + (7500\Omega \| 1450\Omega) = 2220\Omega$$

$$\frac{1}{R_{1s}C_1} = \frac{1}{2.22k\Omega \cdot 2\mu F} = 225 \text{ rad/s}$$

Time Constant Associated with C_2



To find the time constant associated with C_2:



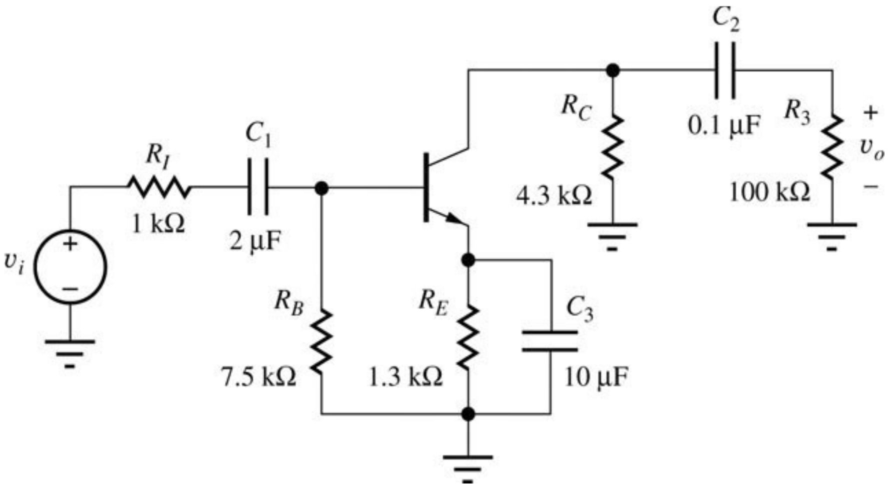
(C_1 and C_3 are short-circuited and set $v_i = 0$)

$$R_{2S} = R_3 + (R_C \| R_{out}^{CE}) = R_3 + (R_C \| r_o) \cong R_3 + R_C$$

$$R_{2s} = 100k\Omega + (4.3k\Omega \,\|\, 44.7k\Omega) = 104k\Omega$$

$$\frac{1}{R_{2s}C_2} = \frac{1}{104k\Omega \cdot 0.1\mu F} = 96.1\,\text{rad/s}$$

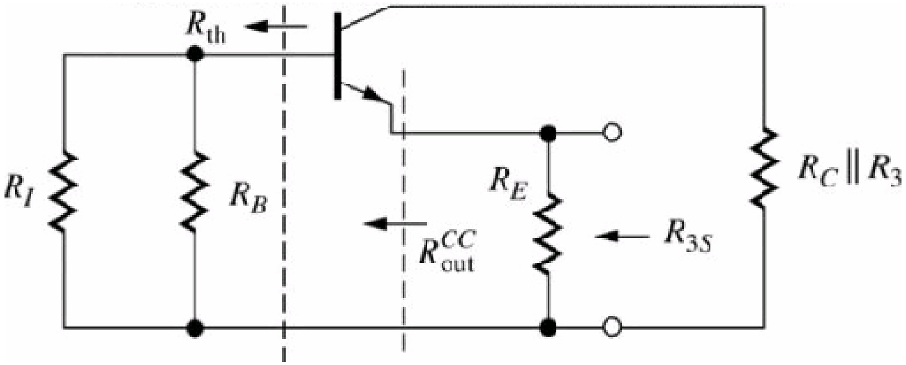
Time Constant Associated with C_3



$$R_{3S} = R_E \| R_{out}^{CC} = R_E \| \frac{r_\pi + R_{th}}{\beta + 1}$$

$$= R_E \| \frac{r_\pi + (R_I \| R_B)}{\beta + 1}$$

To find the time constant associated with C_3:



$$R_{3s} = 1300k\Omega \,\|\, \frac{1450V + 1000\Omega \,\|\, 7500\Omega}{101}$$

$$= 22.7\Omega$$

$$\frac{1}{R_{3s}C_2} = \frac{1}{22.7k\Omega \cdot 10\mu F} = 4410 \text{ rad/s}$$

(C_1 and C_2 are short-circuited and set $v_i = 0$)

Lower Cutoff Frequency

The lower cutoff frequency is:

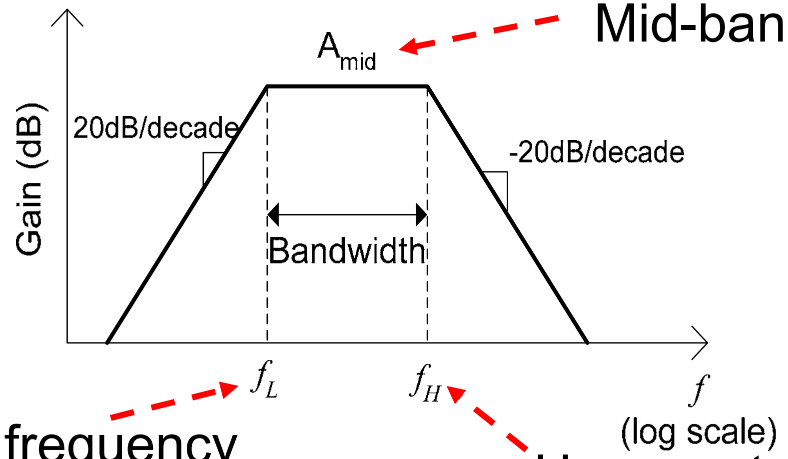
$$\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{iS} C_i} = 225 + 96.1 + 4410 = 4730 \; rad/s$$

and

$$f_L = \frac{\omega_L}{2\pi} = 753\,\mathrm{Hz}$$

In this example the time constant associated with the bypass capacitor C_3 is dominant.

High Frequency Response



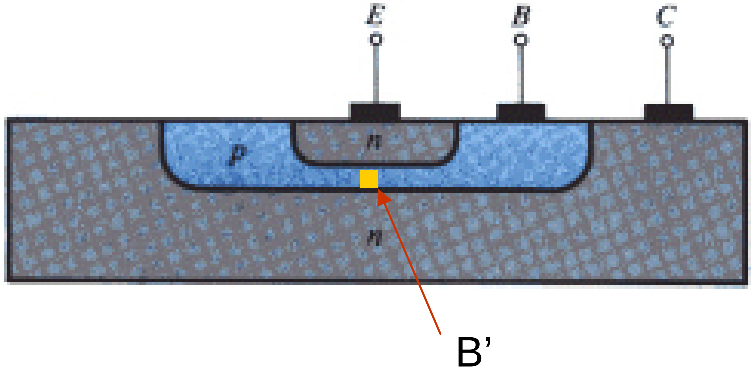
Mid-band gain

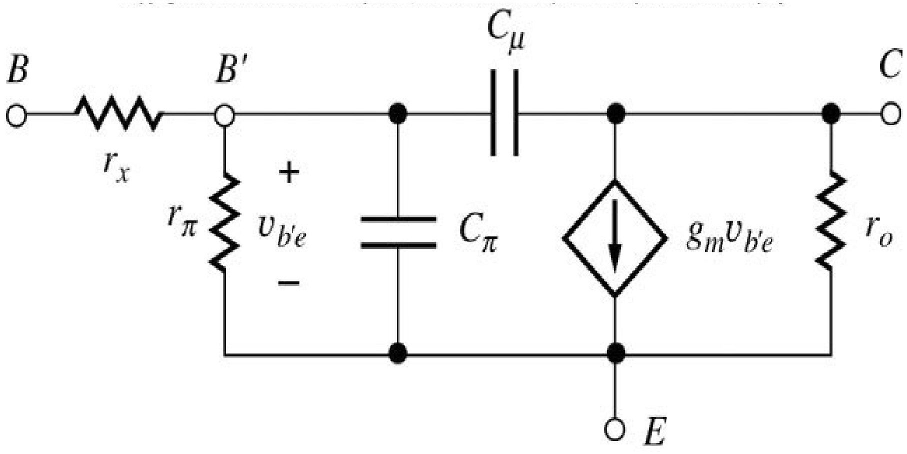
Lower cut-off frequency

Upper cut-off frequency

- At high frequency side, drop in gain is caused by transistor’s parasitic capacitors

High Frequency Small Signal Model for BJT



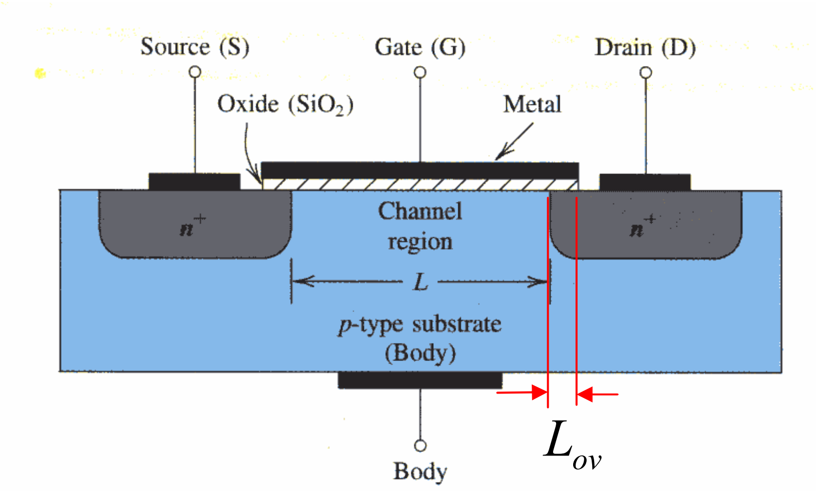


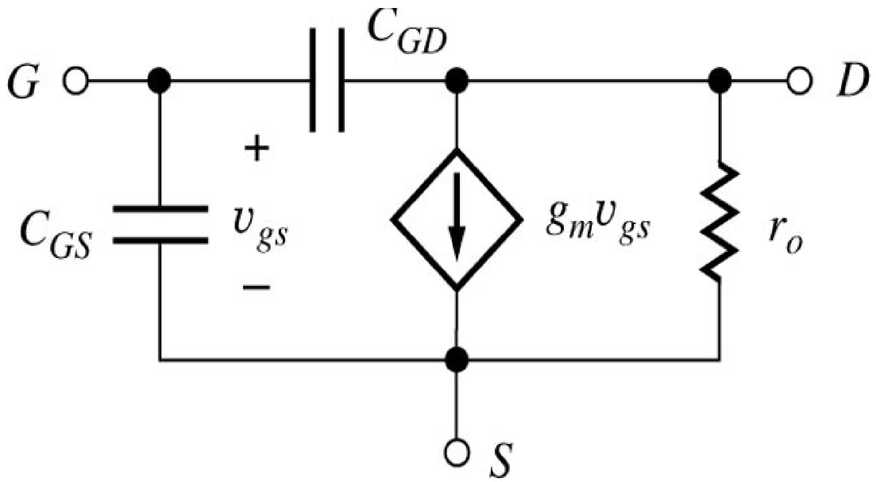
Model for active mode BJT

C_π: diffusion capacitance of the forward-biased base-emitter junction.
C_μ: depletion capacitance of the reverse-biased base-collector junction.

r_x: the resistance of the silicon material of the base region between the base terminal and the intrinsic base terminal B’ that is right under the emitter region.

High-frequency Small Signal Model for MOSFET



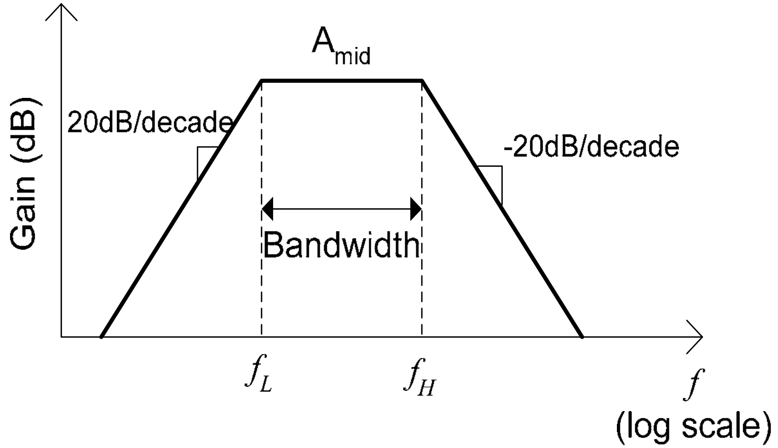


Model for saturation mode MOSFET

$C_{gs} = \frac{2}{3} WLC_{ox}$ = the capacitance between the Gate and the conducting channel.

$C_{gd} = C_{ov} = WL_{ov}C_{ox}$ = the overlap capacitance (very small).

Open-Circuit Time Constant Method to Determine f_H

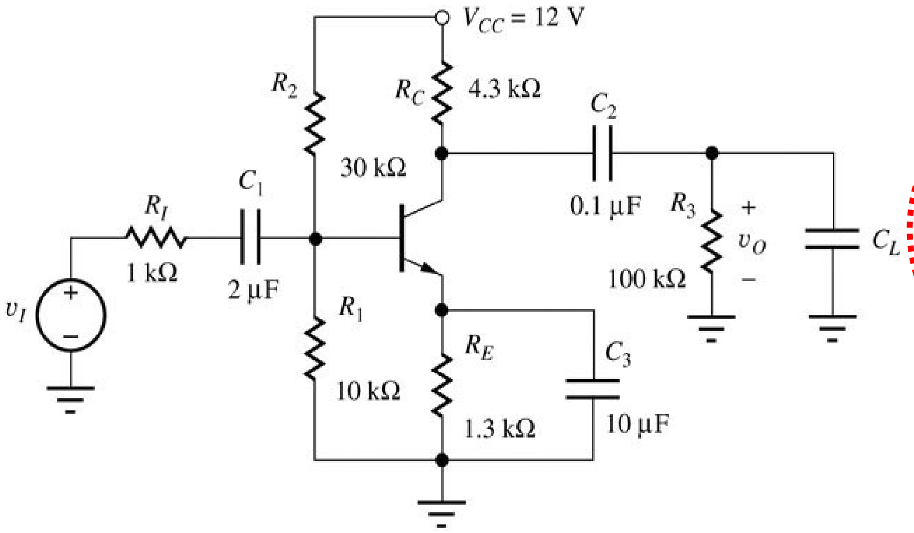


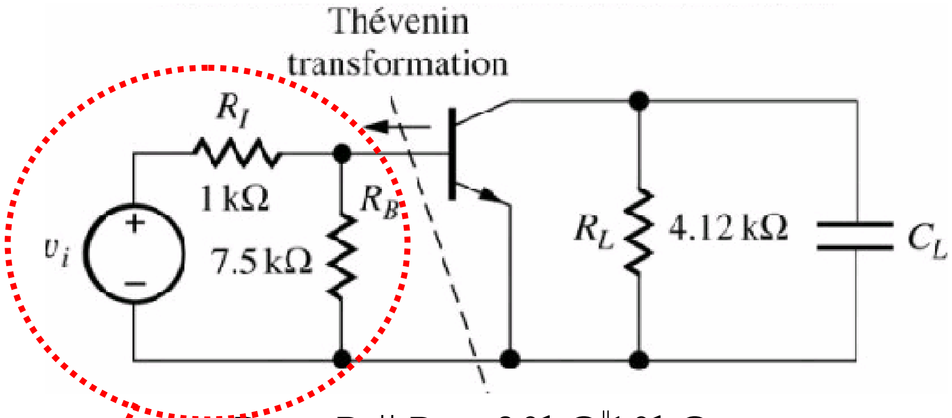
f_H can be estimated by open-circuit time constant method:

$$\boxed{\omega_H \cong \frac{1}{\sum_{i=1}^{m} R_{io} C_i}, \qquad f_H = \omega_H / 2\pi}$$

where R_{io} is resistance at terminals of i^{th} capacitor C_i with all other capacitors open-circuited.

High Frequency Analysis of C-E Amplifier



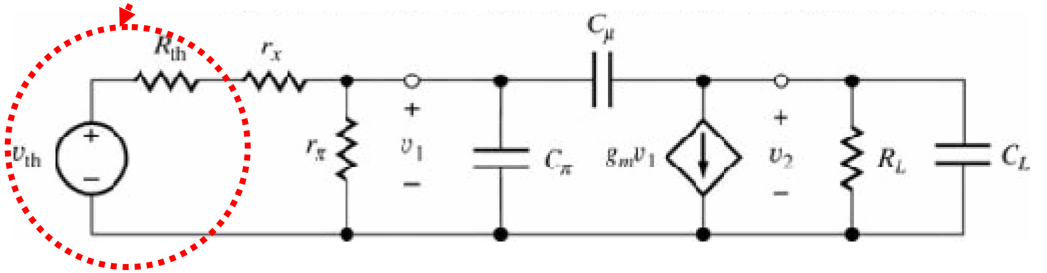


$$R_B = R_1 \| R_2 = 30\text{k}\Omega \| 10\text{k}\Omega$$

$$R_L = R_3 \| R_C = 100\text{k}\Omega \| 4.3\text{k}\Omega$$

β= 100 and V_A=75V

Let Q-point be (1.6mA, 3V)



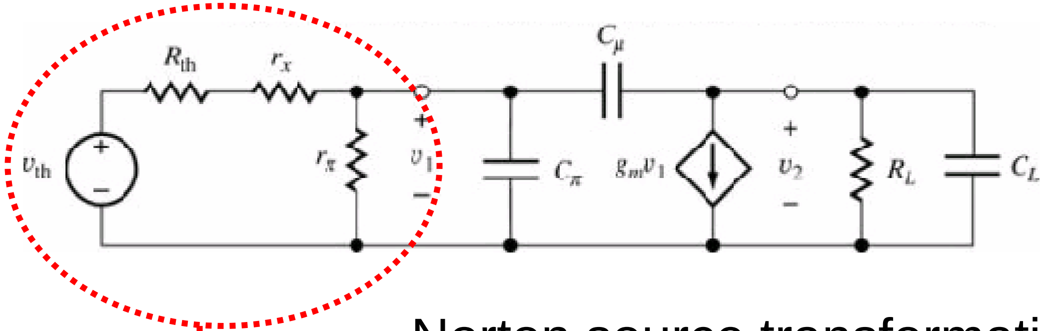
$$C_\pi = 19.9pF$$

$$C_\mu = 0.5pF$$

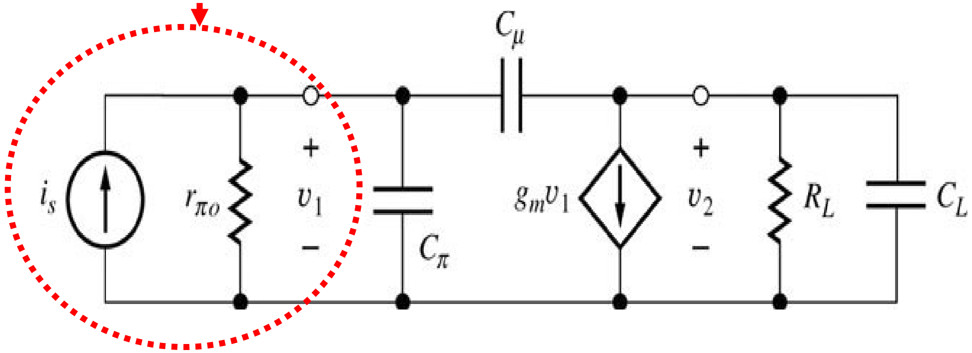
$$r_x = 250\Omega$$

$$v_{th} = v_i \frac{R_B}{R_I + R_B} \qquad R_{th} = \frac{R_I R_B}{R_I + R_B} = 882\Omega$$

High Frequency Small Signal Equivalent



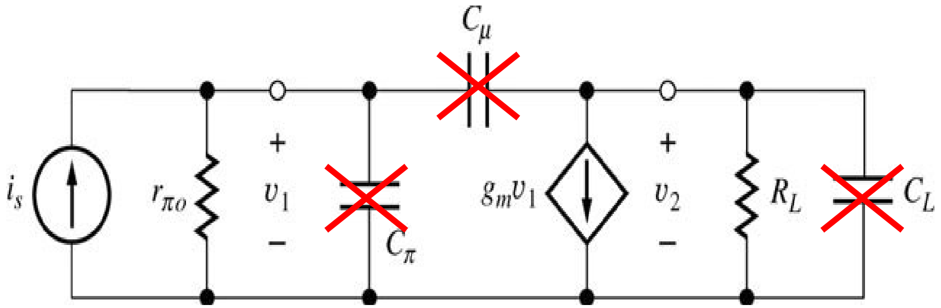
Norton source transformation



$$i_s = \frac{v_{th}}{R_{th} + r_x}$$

$$r_{\pi o} = r_\pi \| (R_{th} + r_x)$$

Determine A_{mid}



$$v_2 = -g_m (i_s r_{\pi 0}) R_L$$

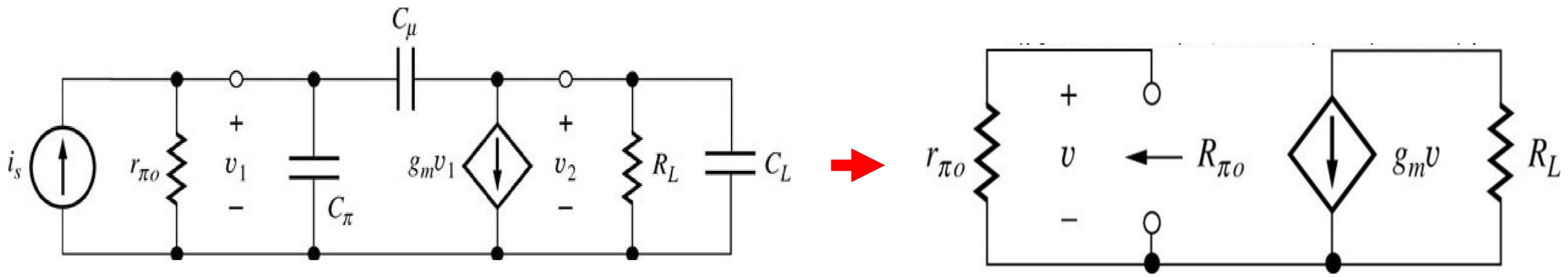
$$i_s = \frac{v_{th}}{R_{th} + r_x} \qquad r_{\pi o} = r_\pi \| (R_{th} + r_x)$$

$$\frac{v_2}{v_{th}} = -g_m R_L \frac{r_{\pi 0}}{R_{th} + r_x} = -g_m R_L \frac{r_\pi}{R_{th} + r_x + r_\pi}$$

$$A_{mid} = -\frac{\beta_o R_L}{R_{th} + r_x + r_\pi} = -\frac{100(4120)}{882 + 250 + 1560} = -153$$

OCTC: Time Constant Associated with C_π

To find the time constant associated with C_π:



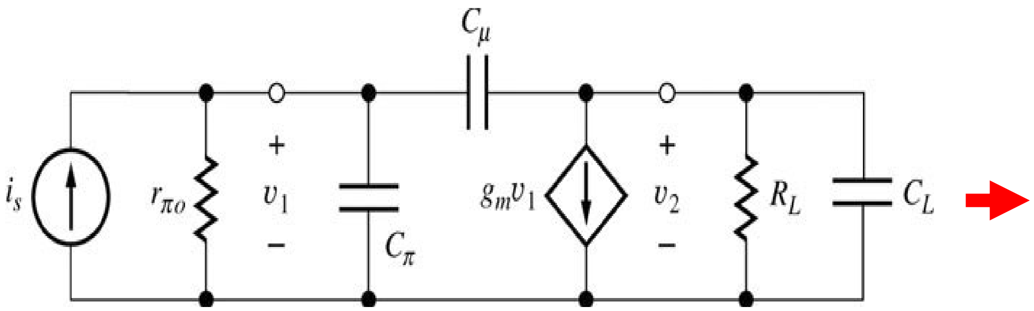
(C_μ is open-circuited and set $i_s = 0$)

$$R_{\pi o} = r_{\pi o} = r_\pi \,||\, (R_{th} + r_x) = 1.56k\Omega \,||\, (882\Omega + 250\Omega) = 656\Omega$$

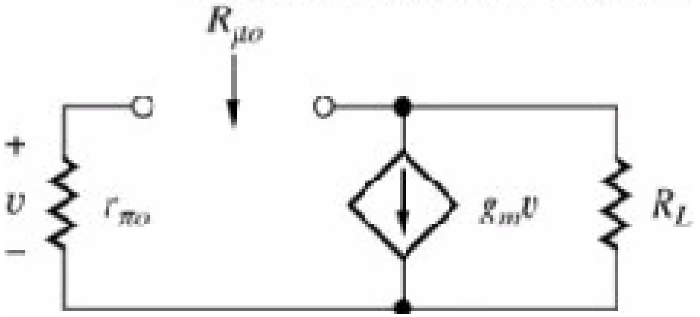
$$C_\pi = 19.9\,pF$$

$$C_\pi R_{\pi 0} = 1.3 \times 10^{-8}$$

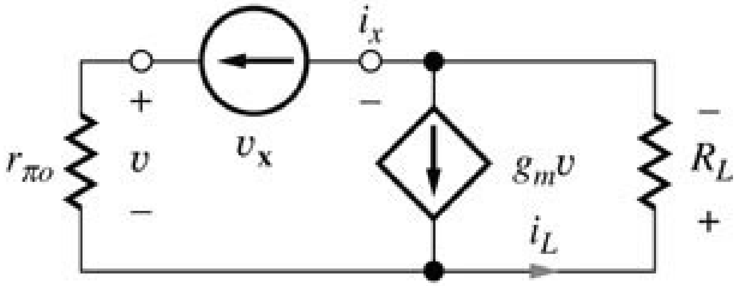
OCTC: Time Constant Associated with C_μ



To find the time constant associated with C_μ:



(C_π is open-circuited and set $i_s = 0$)



$$v_x = i_x r_{\pi 0} + i_L R_L = i_x r_{\pi 0} + (i_x + g_m v) R_L$$

$$R_{\mu o} = \frac{v_x}{i_x} = r_{\pi o}\left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right)$$

$$v = i_x r_{\pi 0}$$

$$\therefore R_{\mu o} = 656\Omega\left(1 + 0.064(4120) + \frac{4120}{656}\right) = 178k\Omega$$

$$C_\mu R_{\mu 0} = 89.0 \times 10^{-9}$$

Upper Cutoff Frequency

Upper cutoff frequency:

$$\omega_H \cong \frac{1}{R_{\pi o}C_\pi + R_{\mu o}C_\mu} = \frac{1}{1.3\times10^{-8} + 89\times10^{-9}} = 9.8\times10^6 \text{ rad/s}$$

$$f_H = \frac{\omega_H}{2\pi} = 1.56\,\text{MHz}$$