Chapter 16

R,*L*, and *C* Elements and the Impedance Concept

Introduction

- To analyze ac circuits in the time domain is not very practical
- It is more practical to:
  - Express voltages and currents as phasors
  - Circuit elements as impedances
  - Represent them using complex numbers

Introduction

- AC circuits
  - Handled much like dc circuits using the same relationships and laws

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Complex Number Review

- A complex number has the form:
  - $a + jb$, where $j = \sqrt{-1}$ (mathematics uses i to represent imaginary numbers)
  - a is the real part
  - jb is the imaginary part
  - Called rectangular form

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Complex Number Review

- Complex number
  - May be represented graphically with *a* being the horizontal component
  - *b* being the vertical component in the complex plane

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Conversion between Rectangular and Polar Forms

- If **C** = $a + jb$ in rectangular form, then **C** = $C\angle\theta$, where

$$a = C\cos\theta$$

$$b = C\sin\theta$$

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

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Complex Number Review

- $j^0 = 1$
- $j^1 = j$
- $j^2 = -1$
- $j^3 = -j$
- $j^4 = 1$ (Pattern repeats for higher powers of *j*)
- $1/j = -j$

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Complex Number Review

- To add complex numbers
  - Add real parts and imaginary parts separately
- Subtraction is done similarly

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Review of Complex Numbers

- To multiply or divide complex numbers
  - Best to convert to polar form first
- $(A\angle\theta)\bullet(B\angle\phi) = (AB)\angle(\theta + \phi)$
- $(A\angle\theta)/(B\angle\phi) = (A/B)\angle(\theta - \phi)$
- $(1/C\angle\theta) = (1/C)\angle{-\theta}$

Review of Complex Numbers

- Complex conjugate of $a + jb$ is $a - jb$
- If $C = a + jb$
  - Complex conjugate is usually represented as C^*

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Voltages and Currents as Complex Numbers

- AC voltages and currents can be represented as phasors
- Phasors have magnitude and angle
  - Viewed as complex numbers

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Voltages and Currents as Complex Numbers

- A voltage given as 100 sin (314*t* + 30°)
  - Written as $100\angle 30°$
- RMS value is used in phasor form so that power calculations are correct
- Above voltage would be written as $70.7\angle 30°$

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Voltages and Currents as Complex Numbers

- We can represent a source by its phasor equivalent from the start
- Phasor representation contains information we need except for angular velocity

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Voltages and Currents as Complex Numbers

- By doing this, we have transformed from the time domain to the phasor domain
- KVL and KCL
  - Apply in both time domain and phasor domain

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Summing AC Voltages and Currents

- To add or subtract waveforms in time domain is very tedious
- Convert to phasors and add as complex numbers
- Once waveforms are added
  - Corresponding time equation of resultant waveform can be determined

Important Notes

- Until now, we have used peak values when writing voltages and current in phasor form
- It is more common to write them as RMS values

Important Notes

- To add or subtract sinusoidal voltages or currents
  - Convert to phasor form, add or subtract, then convert back to sinusoidal form
- Quantities expressed as phasors
  - Are in phasor domain or frequency domain

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R,*L*, and *C* Circuits with Sinusoidal Excitation

- *R*, *L*, and *C* circuit elements
  - Have different electrical properties
  - Differences result in different voltage-current relationships
- When a circuit is connected to a sinusoidal source
  - All currents and voltages will be sinusoidal

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R,*L*, and *C* Circuits with Sinusoidal Excitation

- These sine waves will have the same frequency as the source
  - Only difference is their magnitudes and angles

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Resistance and Sinusoidal AC

- In a purely resistive circuit
  - Ohm’s Law applies
  - Current is proportional to the voltage

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Resistance and Sinusoidal AC

- Current variations follow voltage variations
  - Each reaching their peak values at the same time
- Voltage and current of a resistor are in phase

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Inductive Circuit

- Voltage of an inductor
  - Proportional to rate of change of current
- Voltage is greatest when the rate of change (or the slope) of the current is greatest
  - Voltage and current are not in phase

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Inductive Circuit

- Voltage leads the current by 90° across an inductor

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Inductive Reactance

- X_L, represents the opposition that inductance presents to current in an ac circuit
- X_L is frequency-dependent
- $X_L = V/I$ and has units of ohms

$$X_L = \omega L = 2\pi fL$$

Capacitive Circuits

- Current is proportional to rate of change of voltage
- Current is greatest when rate of change of voltage is greatest
  - So voltage and current are out of phase

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Capacitive Circuits

- For a capacitor
  - Current leads the voltage by 90°

Capacitive Reactance

- X_C, represents opposition that capacitance presents to current in an ac circuit
- X_C is frequency-dependent
  - As frequency increases, X_C decreases

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Capacitive Reactance

- $X_C = V/I$ and has units of ohms

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Impedance

- The opposition that a circuit element presents to current is impedance, Z
  - Z = V/I, is in units of ohms
  - Z in phasor form is $Z\angle\theta$
  - θ is the phase difference between voltage and current

Resistance

- For a resistor, the voltage and current are in phase
- If the voltage has a phase angle, the current has the same angle
- The impedance of a resistor is equal to $R\angle 0°$

Inductance

- For an inductor
  - Voltage leads current by 90°
- If voltage has an angle of 0°
  - Current has an angle of -90°
- The impedance of an inductor
  - $X_L \angle 90°$

Capacitance

- For a capacitor
  - Current leads the voltage by 90°
- If the voltage has an angle of 0°
  - Current has an angle of 90°
- Impedance of a capacitor
  - $X_C \angle -90°$

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Capacitance

- Mnemonic for remembering phase
  - Remember ELI the ICE man
- Inductive circuit (L)
  - Voltage (E) leads current (I)
- A capacitive circuit (C)
  - Current (I) leads voltage (E)