Chapter 22

Filters and the Bode Plot

Gain

- Power gain is ratio of output power to input power

$$A_P = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

Gain

- Voltage gain is ratio of output voltage to input voltage

$$A_v = \frac{V_{out}}{V_{in}}$$

3

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

Gain

- Any circuit in which the output signal power is greater than the input signal
  - Power is referred to as an amplifier
- Any circuit in which the output signal power is less than the input signal power
  - Called an attenuator

4

Robbins and Miller

Copyright © 2007 Thomson Delmar Learning

Gain

- Gains are very large or very small
  - Inconvenient to express gain as a simple ratio

Circuit Analysis with Devices:
Theory and Practice
THOMSON DELMAR LEARNING

The Decibel

- Bel is a logarithmic unit that represents a tenfold increase or decrease in power

$$A_{P(\text{bels})} = \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}}$$

Copyright © 2007 Thomson Delmar Learning

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

The Decibel

- Because the bel is such a large unit, the decibel (dB) is often used

$$A_{P(\text{dB})} = 10\log_{10}\frac{P_{\text{out}}}{P_{\text{in}}}$$

7

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

The Decibel

- To express voltage gain in decibels:

$$A_{P(\text{dB})} = 10\log_{10}\frac{P_{\text{out}}}{P_{\text{in}}}$$

$$A_{P(\text{dB})} = 10\log_{10}\frac{V^2_{\text{out}}/R}{V^2_{\text{in}}/R}$$

$$A_{P(\text{dB})} = 10\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2$$

$$A_{P(\text{dB})} = 20\log_{10}\frac{V_{\text{out}}}{V_{\text{in}}} = A_{v(\text{dB})}$$

8

Robbins and Miller

Copyright © 2007 Thomson Delmar Learning

Multistage Systems

- To find total gain of a system having more than one stage, each with a gain of A_n
  - Multiply gains together
  - $A_T = A_1 A_2 A_3 \cdots$

Circuit Analysis with Devices:
Theory and Practice

THOMSON
DELMAR LEARNING

Multistage Systems

- If gains are expressed in decibels (which are logarithmic)
  - Gains will add instead of multiplying
  - $A_{T(dB)} = A_{1(dB)} + A_{2(dB)} + A_{3(dB)} \cdots$

Robbins and Miller

Copyright © 2007 Thomson Delmar Learning

Voltage Transfer Functions

- Ratio of output voltage phasor to input voltage phasor for any frequency
- Amplitude of transfer function is voltage gain

Circuit Analysis with Devices:
Theory and Practice
THOMSON DELMAR LEARNING

Voltage Transfer Functions

- Phase angle θ
  - Represents phase shift between input and output voltage phasors
- If the circuit contains capacitors or inductors
  - Transfer function will be frequency dependent

Copyright © 2007 Thomson Delmar Learning

Transfer Functions

- To examine the operation of a circuit over a wide range of frequencies
  - Draw a frequency response curve
- Any circuit which is said to pass a particular range of frequencies
  - Called a filter circuit

Circuit Analysis with Devices:
Theory and Practice
THOMSON
DELMAR LEARNING

Transfer Functions

- By passing a range of frequencies
  - Filter output response is high enough at these frequencies to be usable
- Common types of filters
  - Low-pass, high-pass, band-pass, and band-reject filters

Robbins and Miller
Copyright © 2007 Thomson Delmar Learning

THOMSON DELMAR LEARNING

Low-Pass Filter

- Has a greater gain at low frequencies
  - At higher frequencies the gain decreases
- Cutoff frequency
  - Occurs when gain drops to ½ power point
  - This is 0.707 of the maximum voltage gain
- At cutoff
  - Voltage gain is –3dB; phase angle is –45°

Circuit Analysis with Devices:
Theory and Practice

THOMSON DELMAR LEARNING

Bode Plots

- A Bode plot is a straight-line approximation to the frequency response of a particular filter
- Abscissa will be the frequency in Hz on a logarithmic scale (base 10)

Robbins and Miller

Copyright © 2007 Thomson Delmar Learning

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

Bode Plots

- Ordinate will be gain in dB on a linear scale
- Asymptotes
  - Actual response will approach the straight lines of the Bode approximation

17

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

Bode Plots

- A decade represents a tenfold increase or decrease in frequency
- An octave represents a two-fold increase or decrease in frequency

18

Robbins and Miller

Bode Plots

- Slopes are expressed in either dB/decade or dB/octave
- A simple *RC* or *RL* circuit will have a slope of 20 dB/decade or 6 dB/octave

Circuit Analysis with Devices:
Theory and Practice
THOMSON
DELMAR LEARNING

Writing Voltage Transfer Functions

- A properly written transfer function allows us to easily sketch the frequency response of a circuit
- First, determine voltage gain when $\omega = 0$ and $\omega \to \infty$ (approaches infinity)

Copyright © 2007 Thomson Delmar Learning

Writing Voltage Transfer Functions

- Use voltage divider rule to write the general expression for transfer function in terms of the frequency

Circuit Analysis with Devices:
Theory and Practice

THOMSON
DELMAR LEARNING

Writing Voltage Transfer Functions

- Simplify results into a form containing only terms of $j\omega\tau$ or $(1 + j\omega\tau)$
- Determine break frequencies at $\omega = 1/\tau$

Robbins and Miller

Copyright © 2007 Thomson Delmar Learning

Writing Voltage Transfer Functions

- Sketch straight-line approximation by separately considering the effects of each term of transfer function
- Sketch actual response freehand from the approximation

Circuit Analysis with Devices:
Theory and Practice
THOMSON DELMAR LEARNING

The *RC* Low-Pass Filter

- A series *RC* circuit with output taken across capacitor is a low-pass filter
- At low frequencies
  – Reactance is high
  – Output voltage is essentially equal to input

Copyright © 2007 Thomson Delmar Learning

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

The *RC* Low-Pass Filter

- At high frequencies
  - Output voltage approaches zero

25

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

THOMSON DELMAR LEARNING

The *RC* Low-Pass Filter

- By applying voltage divider rule
  - Determine transfer function

$$V_{\text{out}} = \frac{X_C}{R + X_C} V_{\text{in}}$$

$$TF = \frac{1}{1 + j\omega RC}$$

26

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

The *RC* Low-Pass Filter

- The cutoff frequency is

$$\omega_C = \frac{1}{RC}$$

27

Robbins and Miller

Circuit Analysis with Devices: Theory and Practice

The *RL* Low-Pass Filter

- Low-pass filter may be made up of a resistor and an inductor
  - Output taken across the resistor
- Transfer function is

$$TF = \frac{R}{R + X_L} = \frac{R}{R + j\omega L}$$

28

Robbins and Miller

The *RL* Low-Pass Filter

- Cutoff frequency is

$$\omega_C = \frac{R}{L}$$

The *RC* High-Pass Filter

- Simple *RC* circuit with output taken across resistor is a high-pass filter
- Transfer function is given by

$$\mathbf{TF} = \frac{\mathbf{R}}{\mathbf{R} + \mathbf{Z}_C} = \frac{j\omega RC}{1 + j\omega RC}$$

The *RC* High-Pass Filter

- Phase shift is $\theta = 90° - \tan^{-1}(\omega/\omega_c)$
- Cutoff frequency is

$$\omega_c = \frac{1}{RC}$$

The *RL* High-Pass Filter

- *RL* circuit is a high-pass filter if output is taken across the inductor
- Transfer function is

$$\mathbf{TF} = \frac{\mathbf{Z}_L}{\mathbf{R} + \mathbf{Z}_L} = \frac{j\omega L}{R + j\omega L}$$

The *RL* High-Pass Filter

- Cutoff frequency is

$$\omega_c = \frac{R}{L}$$

Circuit Analysis with Devices:
Theory and Practice
THOMSON DELMAR LEARNING

The Band-Pass Filter

- Permits frequencies within a certain range to pass from input to output
- All frequencies outside this range will be attenuated

Robbins and Miller
Copyright © 2007 Thomson Delmar Learning

The Band-Pass Filter

- One way to build a band-pass filter is to cascade a low-pass filter with a high-pass filter
- A band-pass filter can also be constructed from an *RLC* circuit

35

Circuit Analysis with Devices: Theory and Practice
THOMSON DELMAR LEARNING

The Band-Reject Filter

- Passes all frequencies except for a narrow band
- Can be constructed from an *RLC* series circuit
  - Taking output across the inductor and capacitor

36

Robbins and Miller
Copyright © 2007 Thomson Delmar Learning

THOMSON
DELMAR LEARNING

The Band-Reject Filter

- Can also be constructed from a circuit containing a *RC* parallel combination in series with a resistor
  - Taking output across the resistor