Digital Communications I: Modulation and Coding Course

Spring - 2015

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Lecture 4: BandPass Modulation/Demodulation

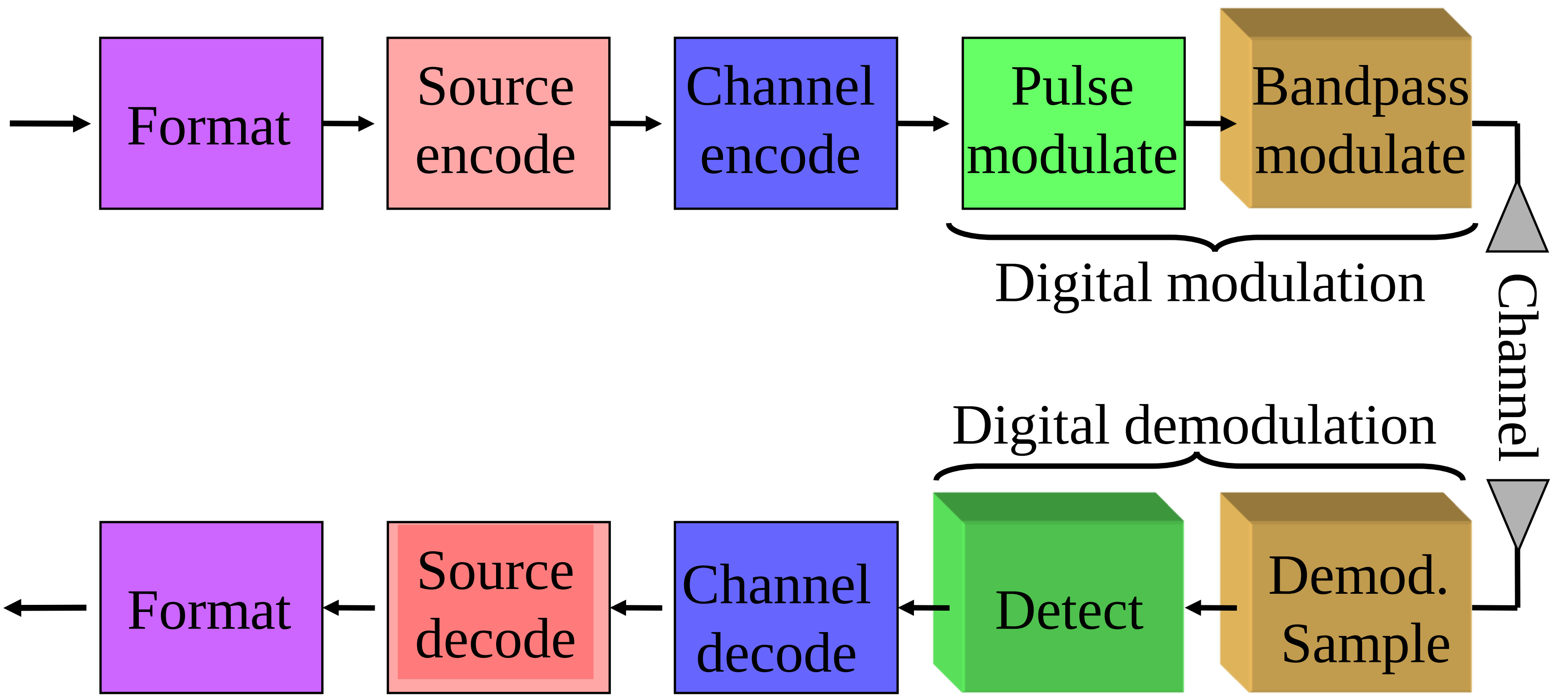
Last time we talked about:

- Another source of error due to filtering effect of the system:
  - Inter-symbol interference (ISI)
- The techniques to reduce ISI
  - Pulse shaping to achieve zero ISI *at the sampling time*
  - Equalization to combat the filtering effect of the channel

Today, we are going to talk about:

- Some bandpass modulation schemes used in DCS for transmitting information over channel
  - M-PAM, M-PSK, M-FSK, M-QAM
- How to detect the transmitted information at the receiver
  - Coherent detection
  - Non-coherent detection

Block diagram of a DCS



Bandpass modulation

- Bandpass modulation: The process of converting a data signal to a sinusoidal waveform where its amplitude, phase or frequency, or a combination of them, are varied in accordance with the transmitting data.
- Bandpass signal:

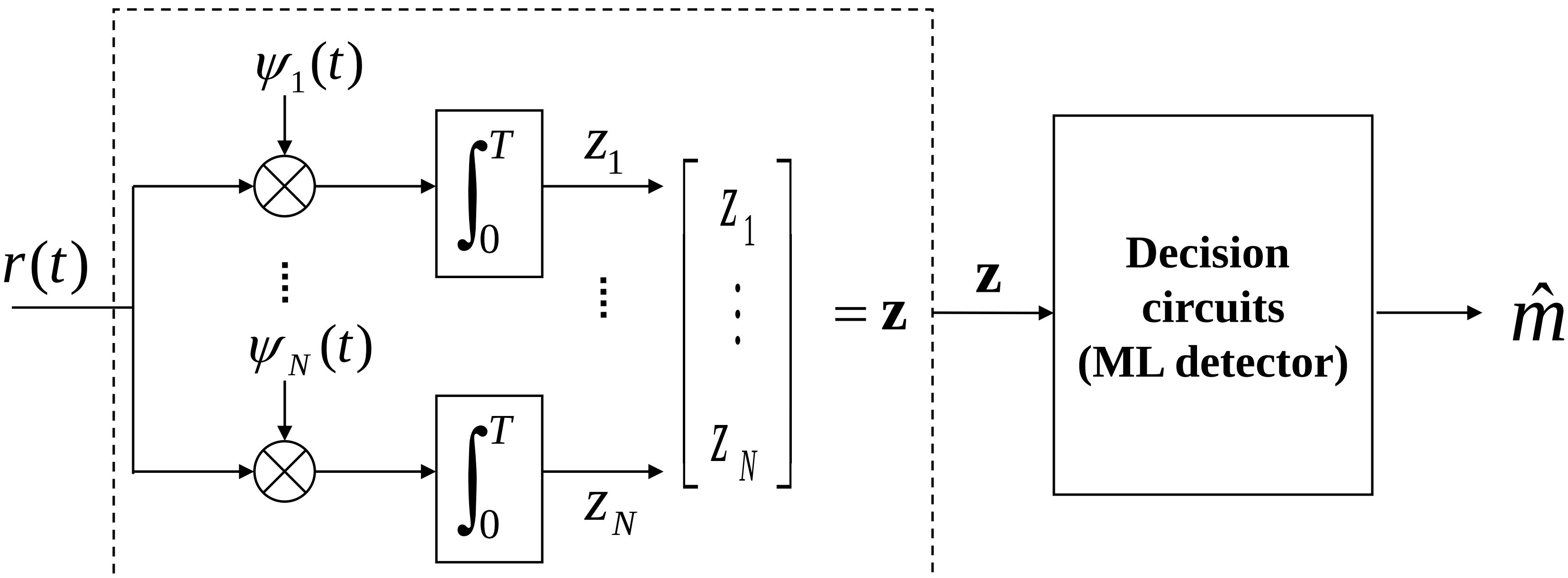
$$s_i(t) = g_T(t)\sqrt{\frac{2E_i}{T}}\cos\left(\omega_c t + (i-1)\Delta\omega t + \phi_i(t)\right) \quad 0 \le t \le T$$

where $g_T(t)$ is the baseband pulse shape with energy .

- We assume here (otherwise will be stated):
  - $g_T(t)$ is a rectangular pulse shape with unit energy. E_g
  - Gray coding is used for mapping bits to symbols.
  - E_s denotes average symbol energy given by $E_s = \frac{1}{M}\sum_{i=1}^{M} E_i$

Demodulation and detection

- **Demodulation:** The receiver signal is converted to baseband, filtered and sampled.
- **Detection:** Sampled values are used for detection using a decision rule such as the ML detection rule.

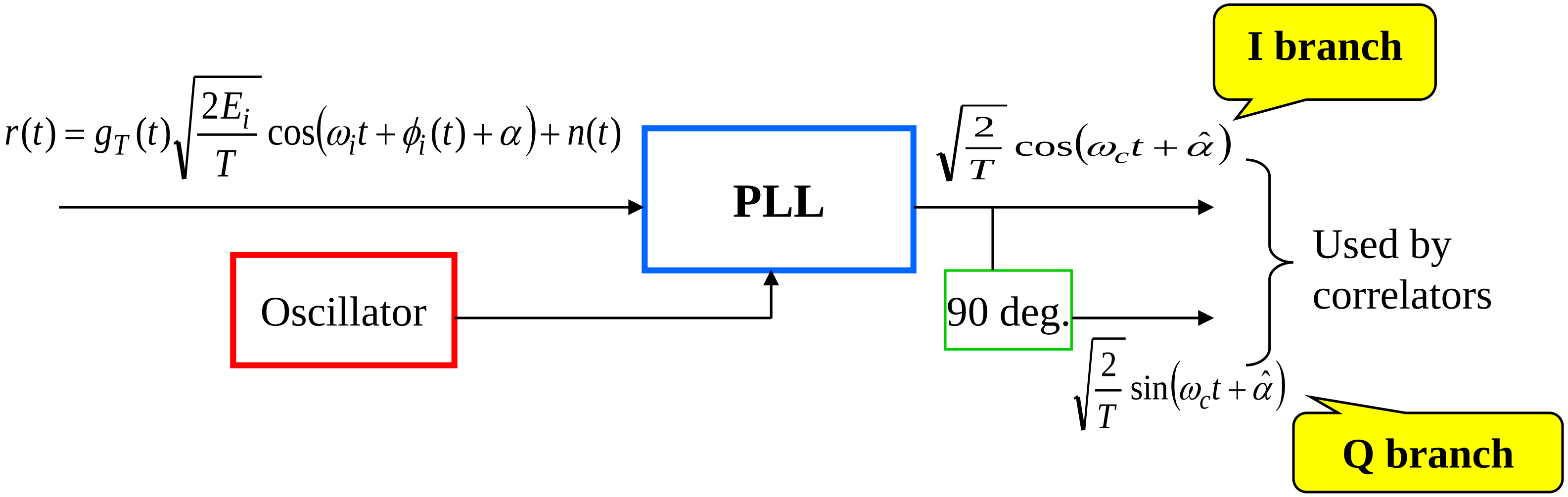


Coherent detection

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Sources of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.

Coherent detection ..

- Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ($\alpha \approx \hat{\alpha}$).



Bandpass Modulation Schemes

- One dimensional waveforms
  - Amplitude Shift Keying (ASK)
  - M-ary Pulse Amplitude Modulation (M-PAM)
- Two dimensional waveforms
  - M-ary Phase Shift Keying (M-PSK)
  - M-ary Quadrature Amplitude Modulation (M-QAM)
- Multidimensional waveforms
  - M-ary Frequency Shift Keying (M-FSK)

One dimensional modulation, demodulation and detection

- Amplitude Shift Keying (ASK) modulation:

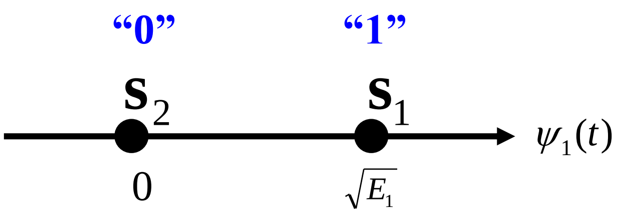
$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \phi)$$

$$s_i(t) = a_i \psi_1(t) \quad i = 1, \ldots, M$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t + \phi)$$

$$a_i = \sqrt{E_i}$$

On-off keying (M=2):



One dimensional mod.,…

- M-ary Pulse Amplitude modulation (M-PAM)

$$s_i(t) = a_i \sqrt{\frac{2}{T}} \cos(\omega_c t)$$

$$s_i(t) = a_i \psi_1(t) \quad i = 1, \ldots, M$$

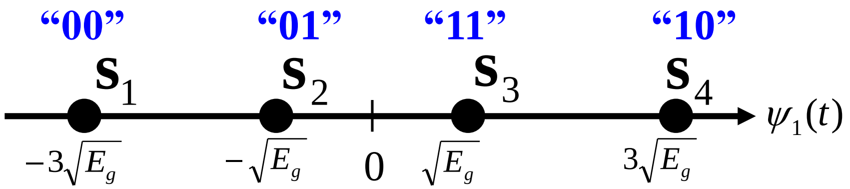
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t)$$

$$a_i = (2i - 1 - M)\sqrt{E_g}$$

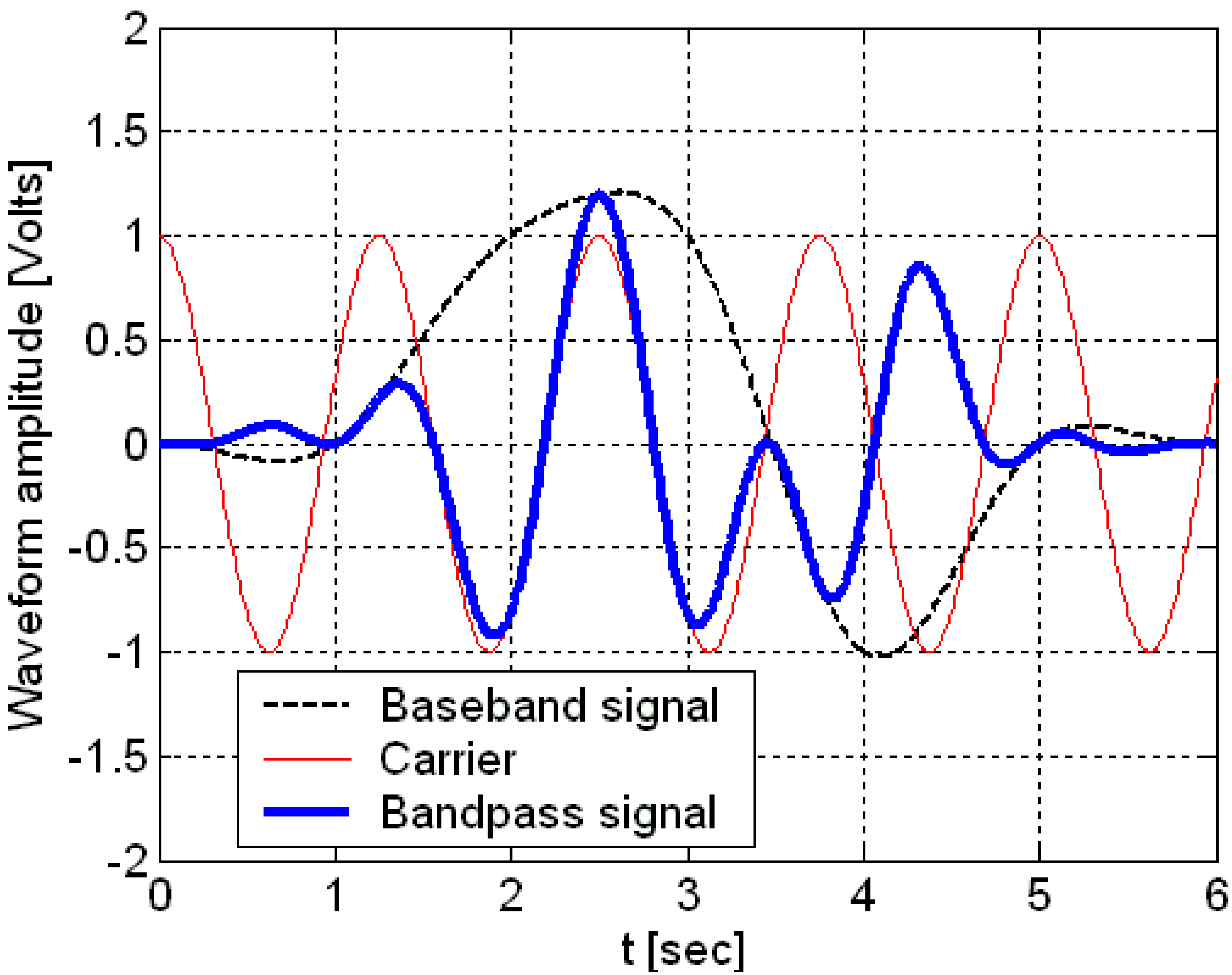
$$E_i = \|\mathbf{s}_i\|^2 = E_g (2i - 1 - M)^2$$

$$E_s = \frac{(M^2 - 1)}{3} E_g$$

4-PAM:

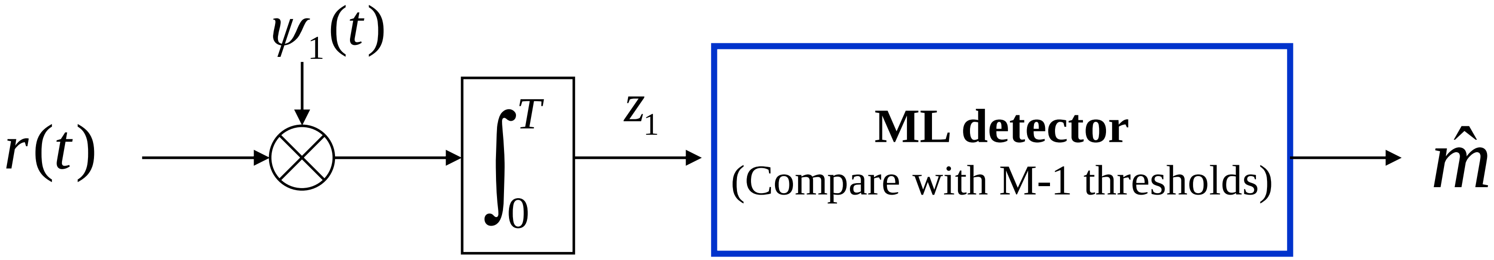


Example of bandpass modulation: Binary PAM



One dimensional mod.,...–cont'd

- Coherent detection of M-PAM



Two dimensional modulation, demodulation and detection (M-PSK)

- M-ary Phase Shift Keying (M-PSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(\omega_c t + \frac{2\pi i}{M}\right)$$

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \ldots, M$$

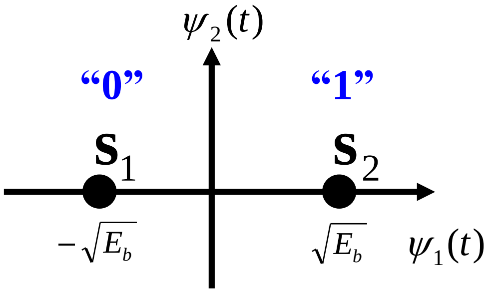
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \qquad \psi_2(t) = -\sqrt{\frac{2}{T}} \sin(\omega_c t)$$

$$a_{i1} = \sqrt{E_s} \cos\left(\frac{2\pi i}{M}\right) \qquad a_{i2} = \sqrt{E_s} \sin\left(\frac{2\pi i}{M}\right)$$

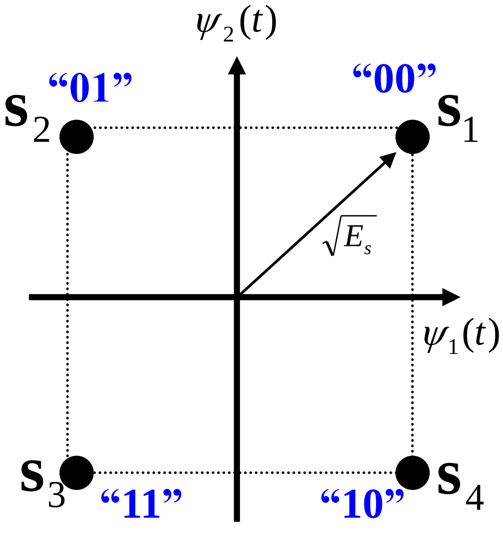
$$E_s = E_i = \|\mathbf{s}_i\|^2$$

Two dimensional mod.,… (MPSK)

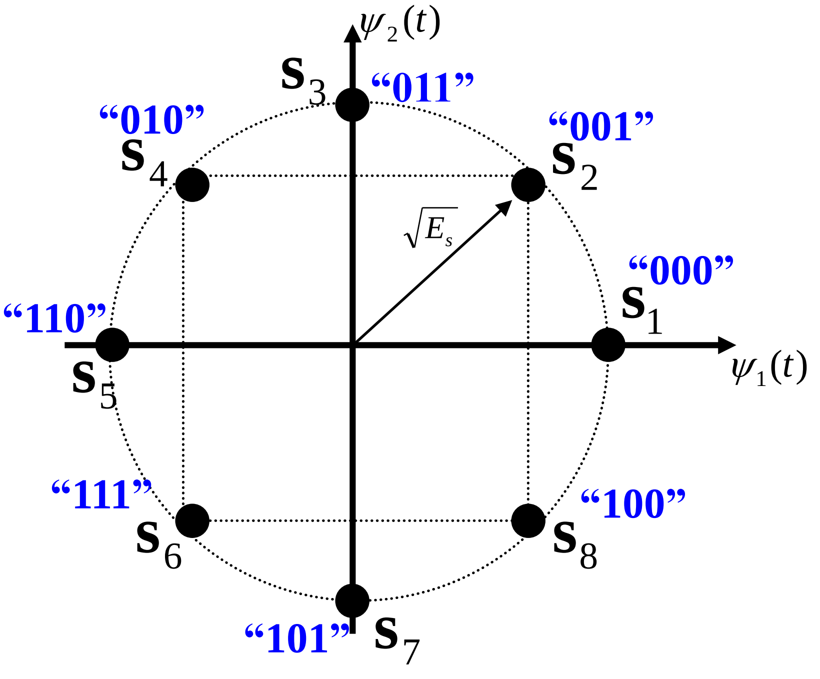
BPSK (M=2)



QPSK (M=4)

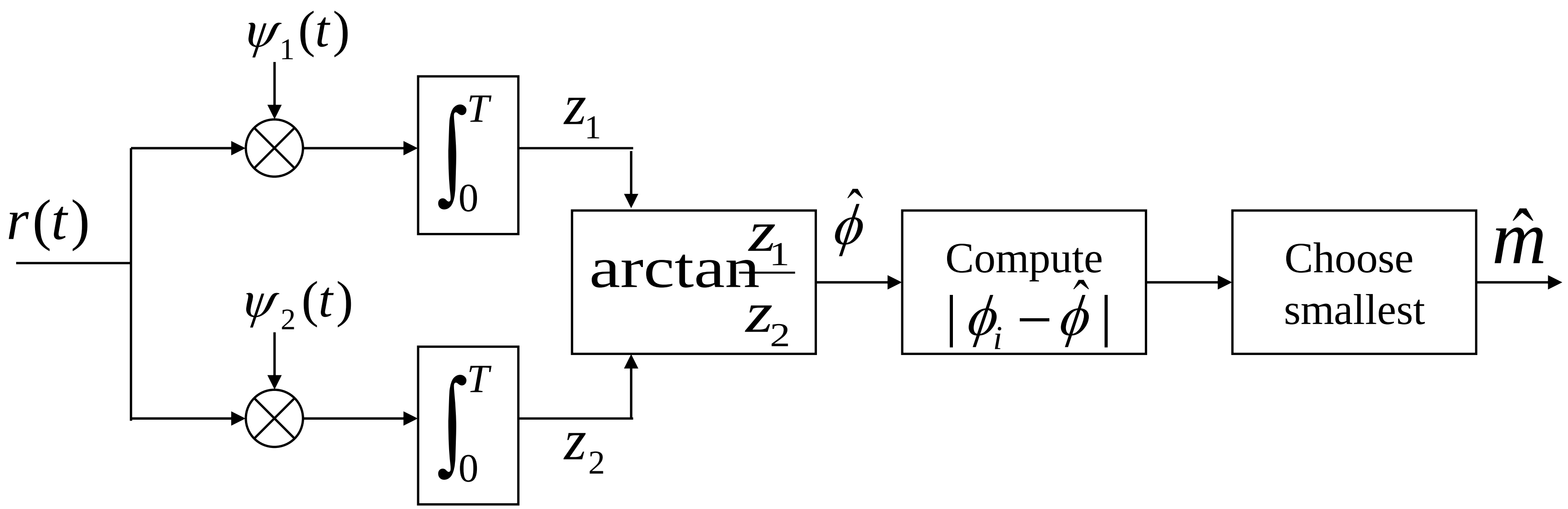


8PSK (M=8)



Two dimensional mod.,...(MPSK)

- Coherent detection of MPSK



Two dimensional mod.,… (M-QAM)

- M-ary Quadrature Amplitude Mod. (M-QAM)

$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \varphi_i)$$

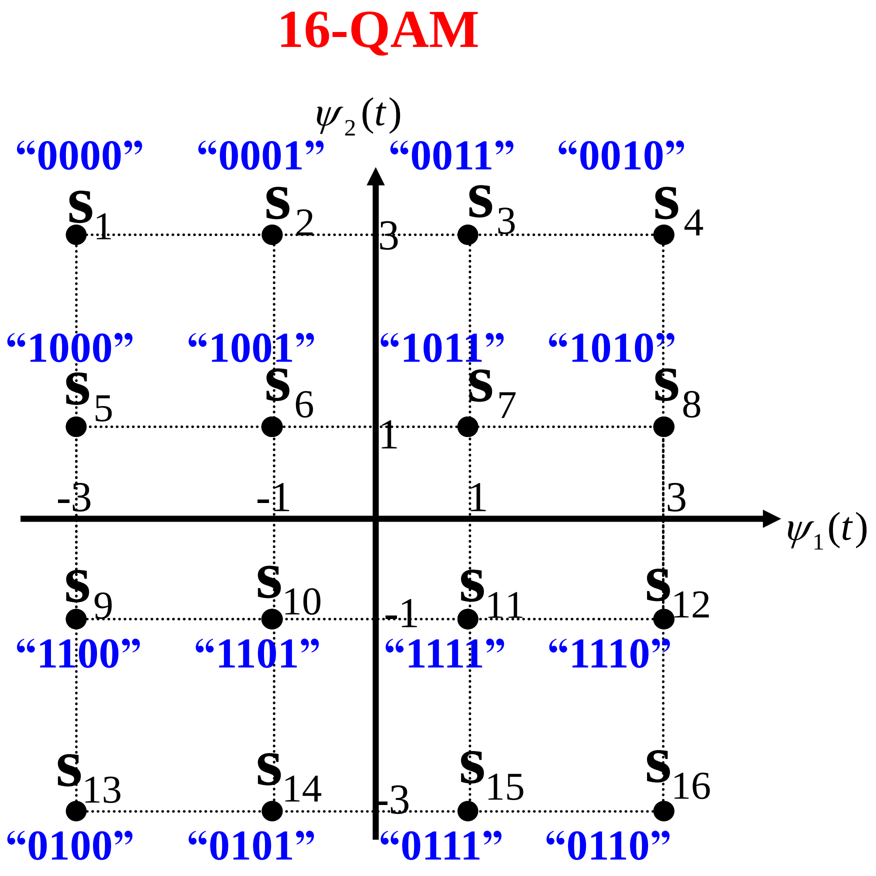
$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \quad i = 1, \ldots, M$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t) \quad \psi_2(t) = \sqrt{\frac{2}{T}} \sin(\omega_c t)$$

where a_{i1} and a_{i2} are PAM symbols and $E_s = \frac{2(M-1)}{3}$

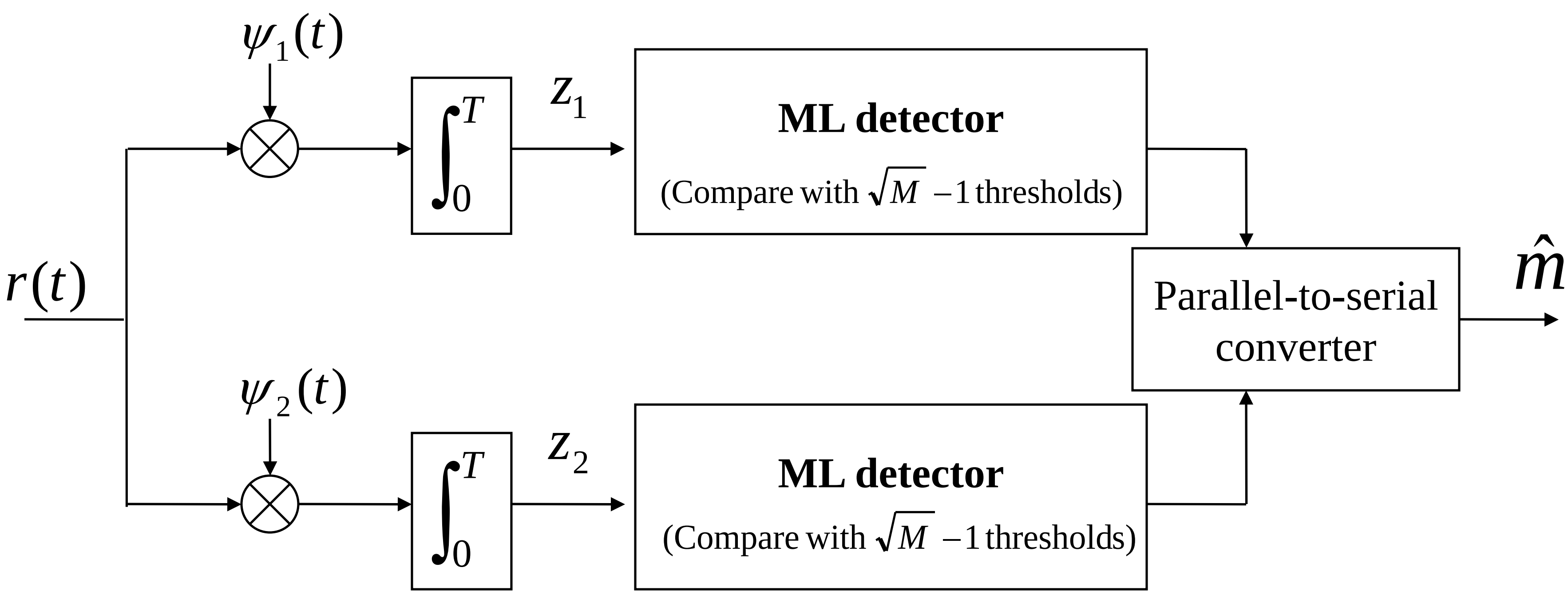
$$(a_{i1}, a_{i2}) = \begin{bmatrix} (-\sqrt{M}+1, \sqrt{M}-1) & (-\sqrt{M}+3, \sqrt{M}-1) & \cdots & (\sqrt{M}-1, \sqrt{M}-1) \\ (-\sqrt{M}+1, \sqrt{M}-3) & (-\sqrt{M}+3, \sqrt{M}-3) & \cdots & (\sqrt{M}-1, \sqrt{M}-3) \\ \vdots & \vdots & \vdots & \vdots \\ (-\sqrt{M}+1, -\sqrt{M}+1) & (-\sqrt{M}+3, -\sqrt{M}+1) & \cdots & (\sqrt{M}-1, -\sqrt{M}+1) \end{bmatrix}$$

Two dimensional mod.,… (M-QAM)



Two dimensional mod.,… (M-QAM)

- Coherent detection of M-QAM



Multi-dimensional modulation, demodulation & detection

- M-ary Frequency Shift keying (M-FSK)

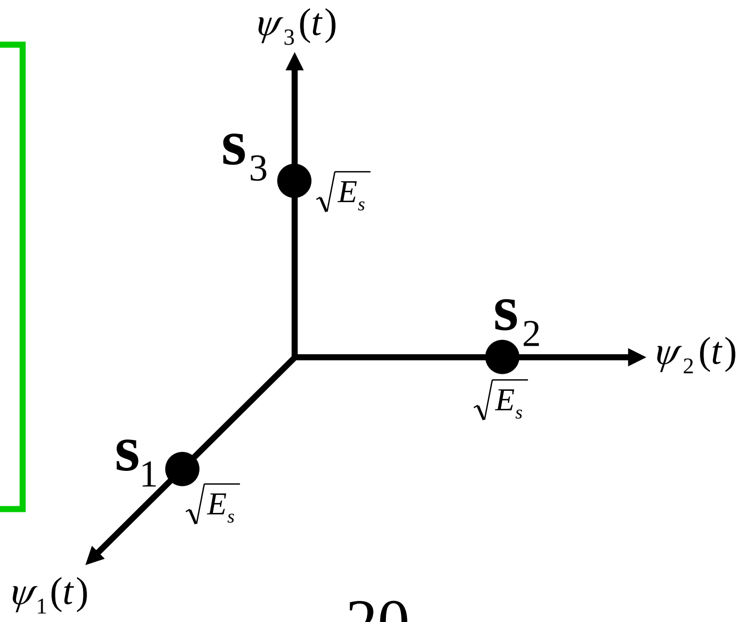
$$s_i(t) = \sqrt{\frac{2E_s}{T}}\cos(\omega_i t) = \sqrt{\frac{2E_s}{T}}\cos(\omega_c t + (i-1)\Delta\omega t)$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{2T}$$

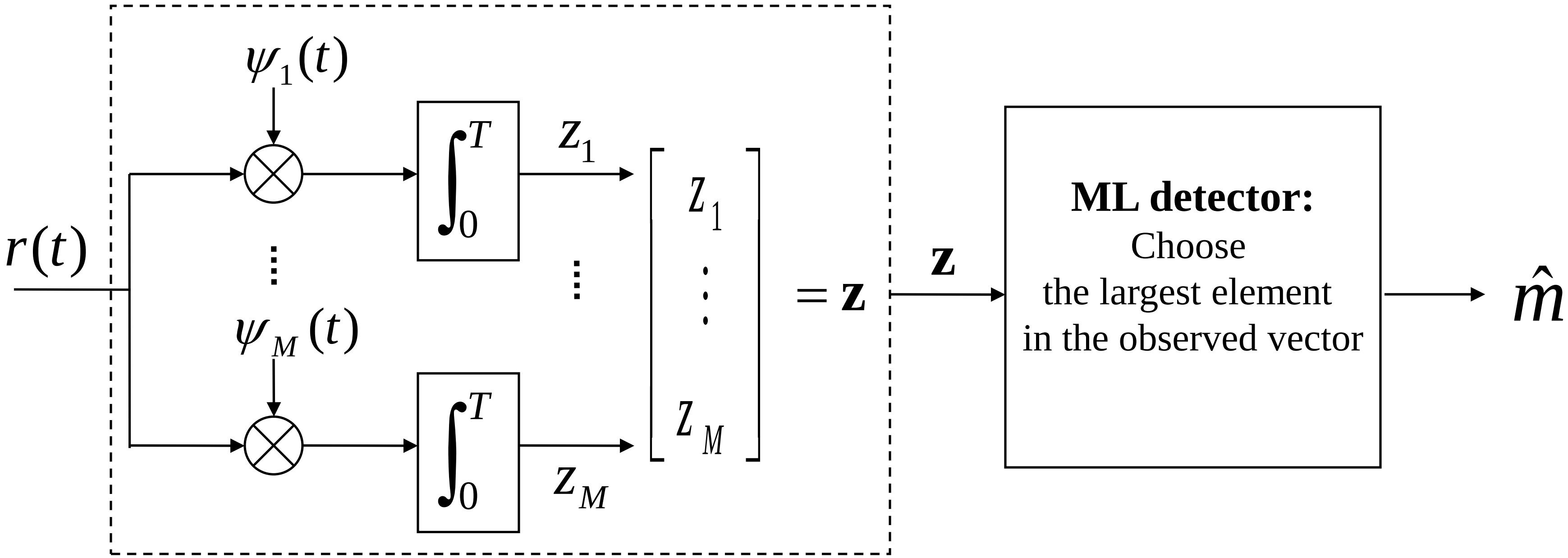
$$s_i(t) = \sum_{j=1}^{M} a_{ij}\psi_j(t) \quad i = 1,\ldots,M$$

$$\psi_i(t) = \sqrt{\frac{2}{T}}\cos(\omega_i t) \qquad a_{ij} = \begin{cases}\sqrt{E_s} & i = j\\ 0 & i \neq j\end{cases}$$

$$E_s = E_i = \|\mathbf{s}_i\|^2$$



Multi-dimensional mod.,…(M-FSK)



Non-coherent detection

- Non-coherent detection:
  - *No need for a reference in phase* with the received carrier
  - *Less complexity* compared to coherent detection at the price of *higher error rate.*

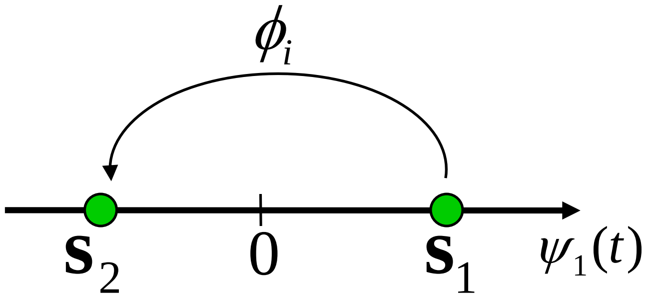
Non-coherent detection …

- Differential coherent detection
  - Differential encoding of the message
    - The symbol phase changes if the current bit is different from the previous bit.

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \theta_i(t)), \quad 0 \le t \le T, \quad i = 1, \ldots, M$$

$$\theta_k(nT) = \theta_k((n-1)T) + \phi_i(nT)$$

Symbol index: k	0	1	2	3	4	5	6	7
Data bits: m_k		1	1	0	1	0	1	1
Diff. encoded bits	1	1	1	0	0	1	1	1
Symbol phase: θ_k	π	π	π	0	0	π	π	π

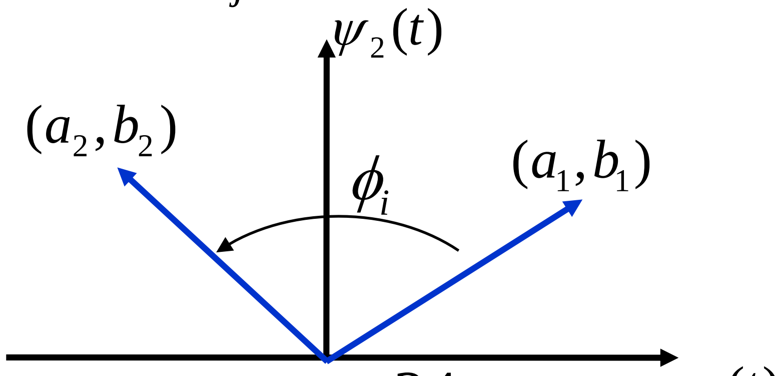


Non-coherent detection …

- Coherent detection for diff encoded mod.
  - assumes slow variation in carrier-phase mismatch during two symbol intervals.
  - correlates the received signal with basis functions
  - uses the phase difference between the current received vector and previously estimated symbol

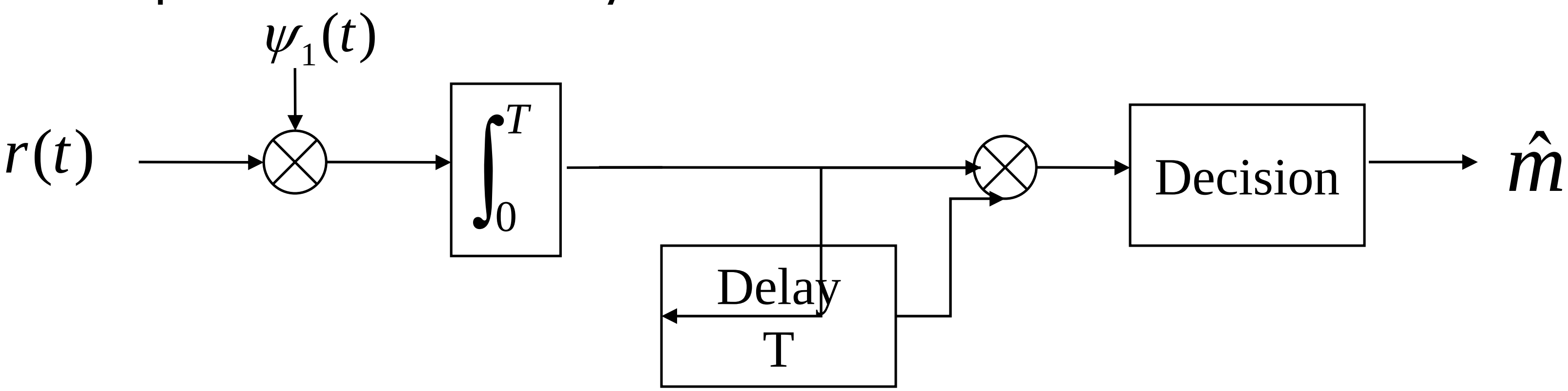
$$r(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_0 t + \theta_i(t) + \alpha\right) + n(t), \quad 0 \le t \le T$$

$$\left(\theta_i(nT) + \alpha\right) - \left(\theta_j((n-1)T) + \alpha\right) = \theta_i(nT) - \theta_j((n-1)T) = \phi_i(nT)$$

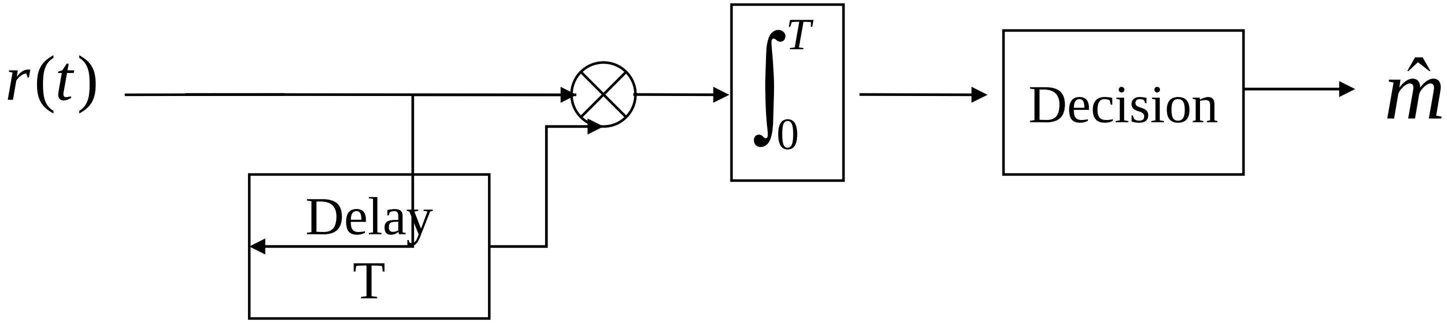


Non-coherent detection …

- Optimum differentially coherent detector



- Sub-optimum differentially coherent detector



  - Performance degradation about 3 dB by using sub-optimal detector

Non-coherent detection …

- Energy detection
  - Non-coherent detection for orthogonal signals (e.g. M-FSK)
    - Carrier-phase offset causes partial correlation between I and Q branches for each candidate signal.
    - The received energy corresponding to each candidate signal is used for detection.

Non-coherent detection ...

- Non-coherent detection of BFSK