EE-387 Probability for Electrical and Computer Engineers

Assignment 1 (due on Thursday, July 14, 2005 before lecture)

Problem 1: A universal set is $S = \{-20 < s \leq -4\}$. If $A = \{-10 \leq s \leq -5\}$ and $B = \{-7 < s < -4\}$, find:

(a) $A \cup B$.

(b) $A \cap B$.

(c) A third set C such that the sets $A \cap C$ and $B \cap C$ are as large as possible while the smallest element in C is -9.

(d) What is the set $A \cap B \cap C$?

(Hint: choose the signs at the boundaries carefully.)

Problem 2: Prove the following version of De Morgan's Law: for any two sets A and B,

$$(A \cap B)^c = A^c \cup B^c.$$

(Hint: do not use the Venn diagram or the duality principle.)

Problem 3: In a high school graduate class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history and neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three subjects. If a student is selected a random, find:

(a) The probability that he takes history and psychology but not mathematics.

(b) The probability that if he is enrolled in history, he takes all three subjects.

(c) The probability that he takes mathematics only.

(Hint: it is helpful to draw a Venn diagram.)

Problem 4: Consider the following variation of the car-goat problem solved in class. This time there are four doors, three goats, and one car. You choose a door at random and then the host selects a door with a goat behind it at random, which he opens. Suppose you switch to one of the other two doors, picking one at random. What is the probability now of winning the car? What is the probability of winning the car if you don't switch?

Problem 5: (Problem 1.6.4 from Yates and Goodman) In an experiment, A, B, C, and D are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

(c) Find $P[D]$, $P[C \cap D^c]$, $P[C^c \cap D^c]$, and $P[C|D]$.

(d) Find $P[C \cup D]$ and $P[C \cup D^c]$.

(e) Are C and D^c independent?

Problem 6: Show that a conditional probability $P[A|B]$ satisfies three probability axioms

(a) $P[A|B] \geq 0$

(b) $P[S|B] = 1$

(c) If $A = A_1 \cup A_2 \cup \ldots$ with $A_i \cap A_j = \phi$, for $i \neq j$, then

$$P[A|B] = P[A_1|B] + P[A_2|B] + \cdots$$

(Hint: you may assume that all sets are contained in the same sample space S.)

Problem 7: (Problem 1.7.6 from Yates and Goodman) A machine produces photo detectors in pairs. Tests show that the first photo detector is acceptable with probability $3/5$. When the first photo detector is acceptable, the second photo detector is acceptable with probability $4/5$. If the first photo detector is defective, the second photo detector is acceptable with probability $2/5$.

(a) What is the probability that exactly one photo detector of a pair is acceptable?

(b) What is the probability that both photo detectors in a pair are defective?

Problem 8: The binary communication system given in class is to be extended to a ternary communication system, i.e., the case of three transmitted symbols 0, 1, 2. Define appropriate events

A_i and B_i, $i = 1,2,3$, to represent symbols after and before the channel, respectively. Assume channel transition probabilities are all equal to $P[A_i|B_j] = 0.1$, $i \neq j$, and are $P[A_i|B_j] = 0.8$ for $i = j = 1,2,3$, while symbol transmission probabilities are $P[B_1] = 0.5$, $P[B_2] = 0.3$, and $P[B_3] = 0.2$.

(a) Sketch a diagram similar to the one given in class.

(b) Compute received symbol probabilities $P[A_1]$, $P[A_2]$, and $P[A_3]$.

(c) Compute the *a posteriori* probabilities for this system.

(d) Determine the decision rule used by a maximum *a posteriori* (MAP) receiver.

Problem 9: (Problem 1.9.1 from Yates and Goodman) Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit.

(a) What is the probability of the code word 00111?

(b) What is the probability that a code word contains exactly three ones?

Problem 10: In a communication system the signal sent from point *a* to point *b* arrives by two paths in parallel. Over each path the signal passes through two repeaters (in series). Each repeater in one path has a probability of failing (becoming an open circuit) of 0.005. This probability is 0.008 for each repeater on the other path. All repeaters fail independently of each other. Find the probability that the signal will not arrive at point *b*.