EE-387 Probability for Electrical and Computer Engineers
Assignment 5 (due 14:00 on Wednesday, August 10, 2005)

Problem 1: (Problem 4.8.7 from Yates and Goodman) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 5x^2/2 & -1 \le x \le 1; 0 \le y \le x^2, \\ 0 & \text{otherwise.} \end{cases}$$

Let $A = \{Y \le 1/4\}$. (a) What is the conditional PDF $f_{X,Y|A}(x,y)$? (b) What is $f_{Y|A}(y)$? (c) What is $E[Y|A]$? (d) What is $f_{X|A}(x)$? (e) What is $E[X|A]$?

Problem 2: (Problem 4.9.4 from Yates and Goodman) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 \le y \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the PDF $f_Y(y)$, the conditional PDF $f_{X|Y}(x|y)$, and the conditional expected value $E[X|Y=y]$.

Problem 3: (Problem 4.11.2 from Yates and Goodman) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = ce^{-(2x^2-4xy+4y^2)}.$$

(a) What are $E[X]$ and $E[Y]$? (b) Find ρ, the correlation coefficient of X and Y. (c) What are $\mathrm{Var}[X]$ and $\mathrm{Var}[Y]$? (d) What is the constant c? (e) Are X and Y independent?

Problem 4: (Problem 4.11.8 from Yates and Goodman) Let X_1 and X_2 have a bivariate Gaussian PDF with correlation coefficient ρ_{12} such that each X_i is a Gaussian random variable with mean μ_i and variance σ_i^2. Show that $Y = X_1X_2$ has variance

$$\mathrm{Var}[Y] = \sigma_1^2\sigma_2^2(1+\rho_{12}^2) + \sigma_1^2\mu_2^2 + \mu_1^2\sigma_2^2 - \mu_1^2\mu_2^2.$$

Hints: Use the iterated expectation to calculate

$$E[X_1^2X_2^2] = E[E[X_1^2X_2^2|X_2]].$$

Problem 5: (Problem 6.2.3 from Yates and Goodman) Random variables X and Y are independent exponential with expected values $E[X] = 1/\lambda$ and $E[Y] = 1/\mu$. If $\mu \neq \lambda$, what is the PDF of $W = X + Y$? If $\mu = \lambda$, what is $f_W(w)$?

Problem 6: (Problem 6.2.5 from Yates and Goodman) Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 8xy & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the PDF of $W = X + Y$?

Problem 7: (a) If X is an Erlang (n, λ) random variable, show that the moment generating function of X is given by

$$\phi_X(s) = \left(\frac{\lambda}{\lambda - s} \right)^n.$$

(b) If X is a Gaussian random variable with mean μ and variance σ^2, show that the moment generating function of X is given by

$$\phi_X(s) = e^{s\mu + s^2\sigma^2/2}.$$

Problem 8: Let X be a Gaussian random variable with mean zero and variance σ^2. Use the moment generating function to show that

$$E[X] = 0, \;\; E[X^2] = \sigma^2, \;\; E[X^3] = 0, \;\; E[X^4] = 3\sigma^4.$$

What can you say about $E[X^n]$ for arbitrary integer values of n?