EE 3025 S2005 Homework Set #1

(due 10:10 AM Friday, January 28, 2005)

Directions: Work all 5 problems. We will choose 3 of the 5 problems randomly for grading.

1. A fair die is rolled 3 times. Describe the sample space S (without listing all the outcomes!). Let A, B, C denote the events

$$\begin{aligned} A &= \{\textit{same number rolled on all three tries}\} \\ B &= \{\textit{sum of the three rolls is } 5\} \\ C &= \{\textit{an odd number is rolled on each try}\} \end{aligned}$$

Find $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$, $P(B \cap C)$, $P(A \cap C)$, $P(A \cap B \cap C)$.

2. Let A, B, C denote events such that $P(A) = 0.57$, $P(B) = 0.49$, $P(C) = 0.13$, $P(A \cap B) = 0.13$, $P(A \cap C) = 0.07$, $P(B \cap C) = 0.04$, $P(A \cap B \cap C) = 0.01$. Compute $P(A \cap B^c \cap C^c)$ and $P(A \cup B^c)$. (Hint: A Venn diagram may help here.)

3. A manufacturer of breakfast cereals implements a quality control program which randomly selects cereal boxes from the production line and tests the contents to see if they snap, crackle, and pop. Let A be the event that the cereal fails to snap, B the event that it fails to crackle, and C the event that it fails to pop. Suppose that $P(A) = 0.25$, $P(B) = 0.2$, $P(C) = 0.25$, $P(A \cap B) = 0.1$, $P(A \cap B \cap C) = 0.05$, $P(A \cap C) = 2P(B \cap C)$, and the probability that at least two of the events A, B, C occur is 0.3.

(a) What is the probability that the cereal passes all three tests, i.e., what is $P(A^c \cap B^c \cap C^c)$?

(b) What is the probability that the cereal fails exactly one of the three tests?

4. Two equally matched baseball teams play a best 4 of 7 championship series. (This means they keep playing games until one of the teams wins 4 games.) Compute the following probabilities:

(a) The probability that the team which wins the series loses the first game.

(b) The probability that the team which wins the series loses the first two games.

(c) The probability that the team which wins the series loses exactly two of the first three games.

5. Bill, Mary, and Joe each have a fair coin and play the following game. First, Bill tosses his coin; if heads comes up, Bill wins the game. If Bill's coin comes up tails, then Mary tosses her coin; if heads comes up, Mary wins the game. If Mary's coin comes up tails, then Joe tosses his coin; if heads comes up, Joe wins the game. If the winner of the game is not determined after all three people have tossed their coins, then the coin tossing is repeated (Bill 1st, Mary 2nd, Joe 3rd) as many times as it is necessary to determine a winner; the winner is the first person to toss heads. The sample space S for this experiment is infinite and can be taken to be

$$S = \{H, TH, TTH, TTTH, TTTTH, TTTTTH, \cdots\}.$$

(a) Write down an infinite series and sum it up to compute the prob that Bill will win the game.

(b) Write down an infinite series and sum it up to compute the prob that Mary will win the game.

(c) Compute the prob that Joe will win the game.

Supplementary Problems: (not to hand in) From the textbook, you can try Problems 1.3.2, 1.4.1, 1.6.4, 1.7.4, 1.9.3.