

EE 3025 S2005 Homework Set #10 Solutions

Mr. AlHussien is grading Problems 2,4

Mr. Msechu is grading Problems 1

Solution to Problem 1.

Solution to (a): Use Section 34.2 of the class notes as a guide:

```
format long
[a,b,c] = solve('a^2+b^2+c^2=8','a*b+b*c=-4','a*c=1');
a=eval(a(1)); b=eval(b(1)); c=eval(c(1));
a,b,c
a =
    2.41421356237309
b =
   -1.41421356237310
c =
    0.41421356237310
```

It looks like the exact solutions would be (one possible choice):

$$a = 1 + \sqrt{2}, \quad b = -\sqrt{2}, \quad c = \sqrt{2} - 1.$$

For future reference, some Chap 11 theory developed week of April 25 will show that if you execute Matlab command

```
conv([a,b,c],[c,b,a])
```

you will get the autocorrelation of the filtered white noise process:

```
a=1+sqrt(2);b=-sqrt(2);c=-1+sqrt(2);
conv([a,b,c],[c,b,a])
ans =
     1         -4         8         -4         1
```

So, theorywise, our filter with tap weights a, b, c as indicated should work. The rest of the solution gives experimental verification of this.

Solution to (b):

```
a=1+sqrt(2);b=-sqrt(2);c=-1+sqrt(2);
z=randn(1,50002);
x=a*z(3:50002)+b*z(2:50001)+c*z(1:50000);
RX0_est=mean(x.^2);
RX1_est=mean(x(2:50000).*x(1:49999));
RX2_est=mean(x(3:50000).*x(1:49998));
RX0_est,RX1_est,RX2_est
RX0_est =
     8.0100
RX1_est =
```

```
-4.0177
RX2_est =
    0.9871
```

Solution to Problem 2.

Solution to (a)(c): Use Experiment 5 of Recitation 12 as a guide:

```
format rat
R=[8 -4 1 0];
%find first order predictor coefficient A
A = R(2)/R(1)
A =
    -1/2

%find second order predictor coefficients B,C
Q2=inv(toeplitz(R(1:2)))*R(2:3)';
B1=Q2(1), B2=Q2(2)
B1 =
    -7/12
B2 =
    -1/6

%find third order predictor coefficients D,E,F
Q3=inv(toeplitz(R(1:3)))*R(2:4)';
C1=Q3(1), C2=Q3(2), C3=Q3(3)
C1 =
    -41/70
C2 =
    -7/40
C3 =
    -1/70
```

Solution to (d): We first estimate the first order MS prediction error in decibels:

```
a=1+sqrt(2);b=-sqrt(2);c=-1+sqrt(2);
z=randn(1,50003);
x=a*z(3:50003)+b*z(2:50002)+c*z(1:50001);
%generate 1st order predictor samples
A=-1/2;
x1hat=A*x;
%estimate decibels for 1st order
x=x(2:50001);
x1hat=x1hat(1:50000);
decibels1=10*log10(8/mean((x-x1hat).^2))
decibels1 =
```

1.2564

You are not required to compare to the actual figure, which is

$$10 \log_{10} \frac{8}{E_1},$$

where

$$E_1 = \frac{R_X(0)^2 - R_X(1)^2}{R_X(0)} = 6.$$

The actual decibel figure is therefore

$$10 \log_{10}(8/6) = 1.2494 \text{ decibels},$$

so our estimate of 1.2564 is pretty accurate.

We now estimate the second order MS prediction error in decibels:

```
a=1+sqrt(2);b=-sqrt(2);c=-1+sqrt(2);
z=randn(1,50004);
x=a*z(3:50004)+b*z(2:50003)+c*z(1:50002);
%generate 2nd order predictor samples
B1=-7/12 ; B2=-1/6;
x2hat=B1*x(2:50002)+B2*x(1:50001);
x2hat=x2hat(1:50000);
x=x(3:50002);
decibels2=10*log10(8/mean((x-x2hat).^2))
decibels2 =
```

1.3821

You are not required to go further. But it is not hard to find the actual decibel figure. First, via the trick used in Section 35.1 of class notes to find first order predictor MS prediction error, you compute the second order MS prediction error as:

$$E[(X_n - \hat{X}_n)X_n] = R_X(0) - B_1R_X(1) - B_2R_X(2) = 35/6.$$

The actual decibel figure is therefore

$$10 \log_{10} \frac{8}{35/6} = 1.3717 \text{ decibels}$$

So, our estimate of 1.3821 decibels is pretty good.

We now estimate the third order MS prediction error in decibels:

```
a=1+sqrt(2);b=-sqrt(2);c=-1+sqrt(2);
z=randn(1,50005);
x=a*z(3:50005)+b*z(2:50004)+c*z(1:50003);
%generate 3rd order predictor samples
C1= -41/70; C2= -7/40; C3 = -1/70;
```

```

x3hat=C1*x(3:50003)+C2*x(2:50002)+C3*x(1:50001);
x3hat=x3hat(1:50000);
x=x(4:50003);
decibels3=10*log10(8/mean((x-x3hat).^2))
decibels3 =

```

1.3928

You are not required to find the actual figure. But here is how you do it. The MS prediction error is

$$R_X(0) - C_1R_X(1) - C_2R_X(2) - C_3R_X(3) = 1633/280.$$

The actual decibel figure is therefore

$$10 \log_{10} \left(\frac{8}{1633/280} \right) = 1.3726 \text{ decibels.}$$

We ran our script for estimating the third order decibel figure several times, and the variance seemed a bit higher than for first order or second order. That is why our estimate does not seem as good. You could instead average up the decibel estimates from a whole bunch of runs; this would give a better estimate.

One can also consider predictors of fourth order, fifth order, etc., but you will not get much more improvement over the 3rd order predictor.

Solution to Problem 3.

Solution to (a):

$$\begin{aligned}
 P[V = U + 1] &= \sum_{k=0}^{\infty} P[U = k, V = k + 1] \\
 &= \sum_{k=0}^{\infty} P[U = k]P[V = k + 1] \\
 &= \sum_{k=0}^{\infty} \frac{\exp(-2)}{k!(k + 1)!} = 0.2153
 \end{aligned}$$

I used the fact that U and V are each Poisson RV's with mean 1.

Solution to (b):

$$\begin{aligned}
 P[V = U + 2] &= \sum_{k=0}^{\infty} P[U = k, V = k + 2] \\
 &= \sum_{k=0}^{\infty} P[U = k]P[V = k + 2] \\
 &= \sum_{k=0}^{\infty} \frac{\exp(-2)}{k!(k + 2)!} = 0.0932
 \end{aligned}$$

Solution to Problem 4.

Solution to (a): As you learned in EE 3015, for any fixed τ in the range $0 < \tau < 1/2$, the signal $f(t + \tau)$ is obtained from the signal $f(t)$ by shifting τ seconds to the LEFT. This gives us the following breakdown of the signal $f(t + \tau)$ for $0 \leq t \leq 1$:

$$f(t + \tau) = \begin{cases} 1, & 0 \leq t < (1/2) - \tau \\ -1, & (1/2) - \tau \leq t < 1 - \tau \\ 1, & 1 - \tau \leq t \leq 1 \end{cases}$$

For this range of τ , we then have

$$R_X(\tau) = \int_0^1 f(t)f(t + \tau)dt,$$

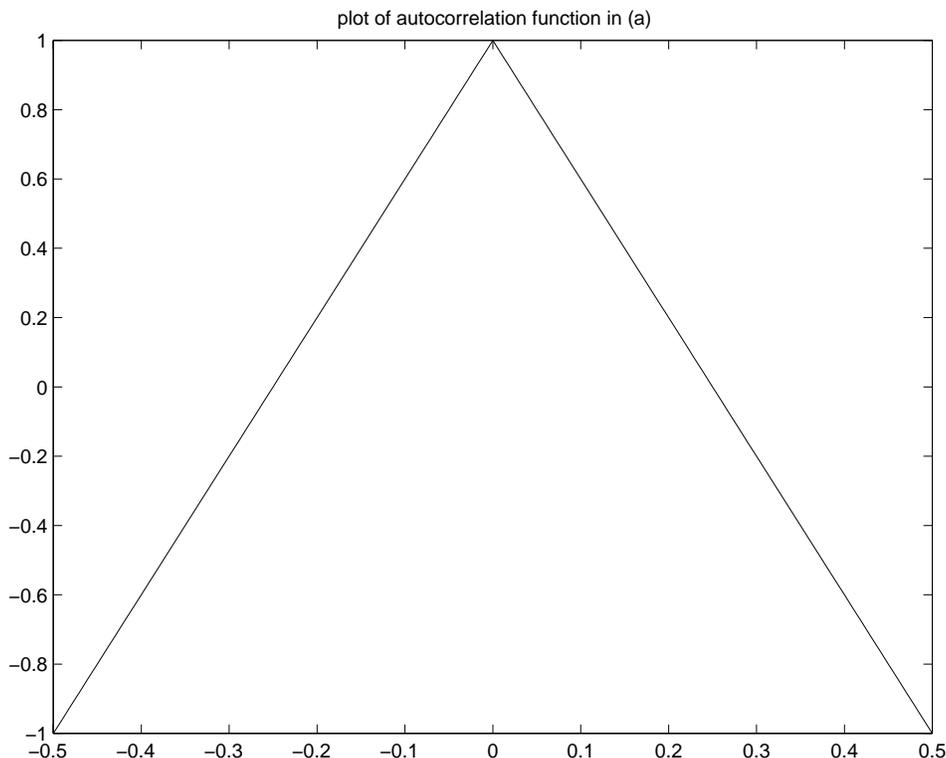
which breaks down as

$$R_X(\tau) = \int_0^{1/2-\tau} dt + \int_{1/2-\tau}^{1/2} (-1)dt + \int_{1/2}^{1-\tau} dt + \int_{1-\tau}^1 (-1)dt = 1 - 4\tau.$$

The other half of $R_X(\tau)$ for negative τ is the reflection of this, since $R_X(\tau)$ is even. Thus,

$$R_X(\tau) = 1 - 4|\tau|, \quad -1/2 \leq \tau \leq 1/2.$$

(For other τ you extend this periodically in both directions, with the period being 1.)



Solution to (b): For $0 < \tau < 1/2$,

$$f(t + \tau) = \begin{cases} t + \tau, & 0 \leq t < 1 - \tau \\ t + \tau - 1, & 1 - \tau \leq t \leq 1 \end{cases}$$

For this range of τ , we then have

$$R_X(\tau) = \int_0^{1-\tau} t(t + \tau)dt + \int_{1-\tau}^1 t(t + \tau - 1)dt = \frac{1}{3} + \frac{\tau^2}{2} - \frac{\tau}{2}.$$

