EE 3025 S2005 Homework Set #2

(due 10:10 AM Friday, February 4, 2005)

Directions: Work all 5 problems. We will grade Problem 1 and will randomly choose two of the other problems for grading.

1. Starting with this homework assignment, there will be one Matlab homework problem assigned per week. This problem is your first Matlab homework problem. On a Matlab homework problem, we expect you, as part of your solution, to turn in a printout of whatever Matlab script(s) you write plus the printout of your Matlab run(s).

In the random experiment for this problem, the output of the experiment is any of the 24 permutations of the 4-tuple $(1, 2, 3, 4)$. The 24 outcomes in the sample space S each have probability 1/24 (equiprobable space). We are interested in the event E that in the outcome (a, b, c, d), no entry is in its right place (that is, $a \neq 1$, $b \neq 2$, $c \neq 3$, $d \neq 4$).

(a) Compute $P(E)$ by hand (not using Matlab). (Hint: If you get stuck see Problem 2.3 of the Chapter 1 Solved Problems.)

(b) Write a Matlab script which can perform n independent trails of the random experiment for any positive integer n.

(c) Run your script 10 times with $n = 5000$. Each time, obtain an empirical estimate of $P(E)$. Let these empirical estimates be $p_1, p_2, \cdots, p_{10}$. Compute the "root mean square deviation"

$$D_{5000} = \sqrt{(1/10)\sum_{i=1}^{10}(p_i - P(E))^2}.$$

(d) Run your script 10 times with $n = 50000$. Each time, obtain an empirical estimate of $P(E)$. Let these empirical estimates be $q_1, q_2, \cdots, q_{10}$. Compute the "root mean square deviation"

$$D_{50000} = \sqrt{(1/10)\sum_{i=1}^{10}(q_i - P(E))^2}.$$

(e) Is D_{50000} smaller than D_{5000}? (Later on in the course, we will have a better idea of how increasing the number of trials improves the empirical probability estimate.)

2. Eight pieces of paper are placed in a box. Each piece has a different three-digit number on it. These numbers are

$$222, 233, 323, 332, 334, 343, 433, 444.$$

A piece of paper is drawn at random from the box and then the number on that piece of paper is recorded. Let E_1, E_2, E_3 be the following events:

$$\begin{aligned} E_1 &= \{\textit{first digit} = (\textit{sum of other two}) - 2\} \\ E_2 &= \{\textit{second digit} = (\textit{sum of other two}) - 2\} \\ E_3 &= \{\textit{third digit} = (\textit{sum of other two}) - 2\} \end{aligned}$$

Prove whether or not the events E_1, E_2, E_3 are independent.

3. Urn A contains 2 white and 3 black balls. Urn B contains 3 white and 5 black balls. Urn C contains 4 white and 2 black balls. A fair coin is tossed. If heads, one ball is selected from Urn A and then one from Urn B. If tails, one ball is selected from Urn B and then one from Urn C.

- **(a)** Find the probability that both balls are the same color.
- **(b)** Given that at least one of the two balls is white, what is the probability that both balls are white?
- **(c)** Given that at least one of the two balls is white, what is the probability that the coin was heads?

4. Hank Kimball (Republican) and Oliver Wendell Douglas (Democrat) were the mayoral candidates in the recent election in the town of Hooterville. Of those who voted, 35% consider themselves Democrats, 25% consider themselves Republicans, and 40% consider themselves Independents. Douglas received 75% of the votes of those who consider themselves Democrats, and half of the votes of those who consider themselves Independents. Kimball received 100% of the votes of those that consider themselves Republicans.

- **(a)** What is the probability that a random voter voted for Douglas?
- **(b)** Given that this randomly selected voter voted for Kimball, what is the probability that this voter considers himself/herself to be a Democrat?

5. Over a certain communication channel, binary symbols can be transmitted and received. When a "0" is transmitted, a "0" is received with probability 0.95 (and therefore a "1" is received with probability 0.05). When a "1" is transmitted, a "1" is received with probability 0.85 (and therefore a "0" is received with probability 0.15).

- **(a)** Suppose "0" and "1" are equally likely to be transmitted. What is the probability that "1" is received?
- **(b)** Find p such that if "0" is transmitted with probability p and "1" is transmitted with probability $1-p$, then "0" and "1" are equally likely to be received.

Supplementary Problems: (not to hand in) From the textbook, you can try Problems 1.10.1, 1.9.5, 1.4.6, 1.5.5, 1.7.8. You can also have a look at the following Solved Problems: Problem 4.4, Problems 5.1-5.6, Problem 7.3, Problems 8.1-8.2. (There are also Relay Circuit Problems; one of these will be on Homework 3.)