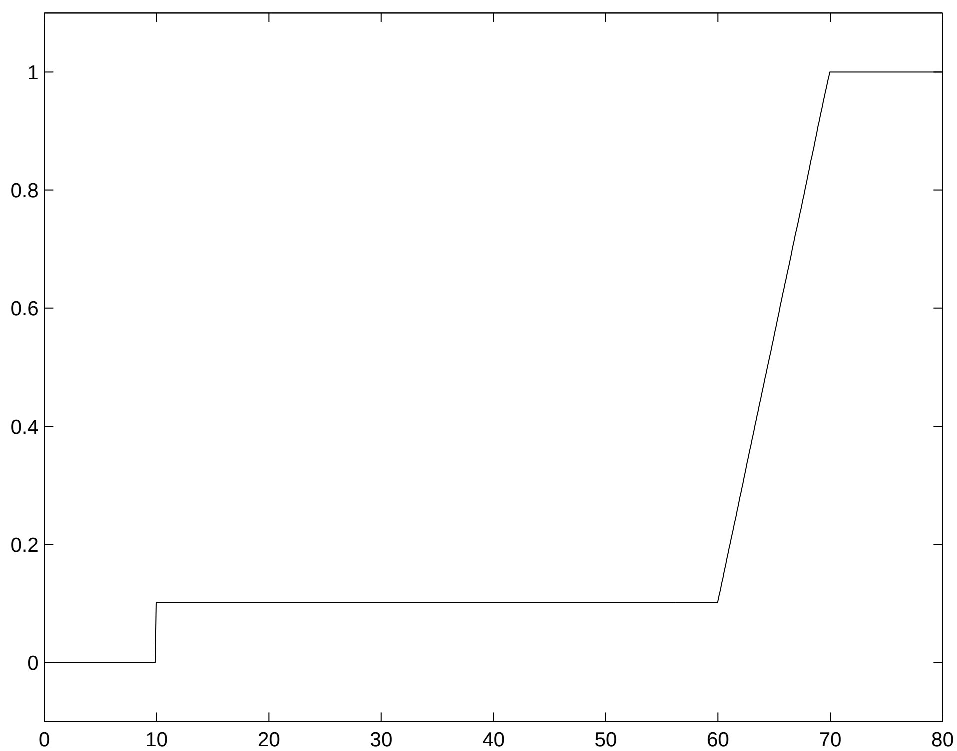
EE 3025 S2005 Homework Set #4 Solutions

Problems 1,3 to be graded by Mr. AlHussien
Problem 5 to be graded by Mr. Msechu
(each graded problem worth 10 POINTS)

Solution to Problem 1.

part(a):

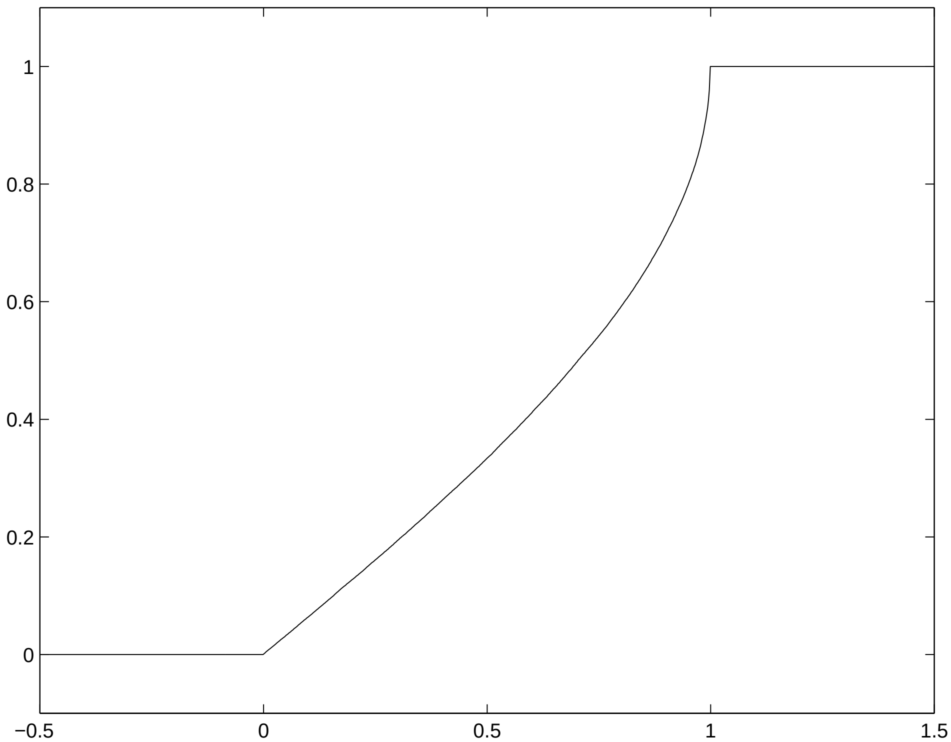
```
x1=10*ones(1,100000); %simulates 100000 throws if foul occurs
x2=10*rand(1,100000)+60; %simulates 100000 throws if foul doesn't occur
b=(rand(1,100000)>0.1); %binary "switch states" (1=nonfoul, 0=foul)
x=b.*x2+(1-b).*x1; %desired samples
```



part(b):

```
x=sin(0.5*pi*rand(1,100000)); %simulated X values
```

The estimated CDF plot is at top of next page.



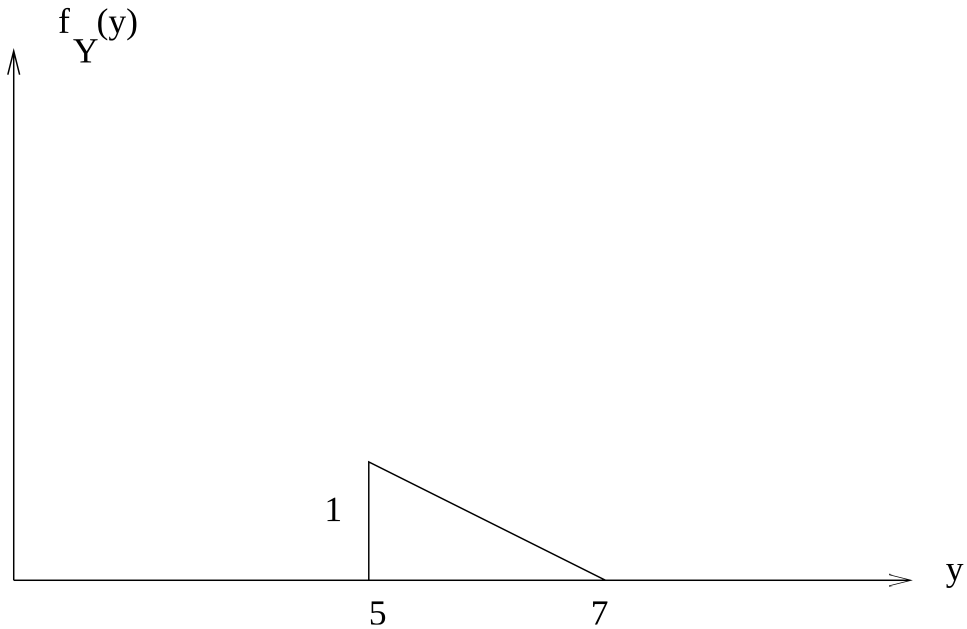
$$\begin{aligned} P(X \leq 0.5) &= P(\sin((0.5)\pi U) \leq 0.5) \\ &= P[U \leq (2/\pi)\mathrm{Sin}^{-1}(0.5)] \\ &= P(U \leq 1/3) = 1/3 \end{aligned}$$

If we measure up to the above curve from the point 0.5 on the horizontal axis, it does seem that we are measuring up a distance of about 0.33.

Solution to Problem 2:
part(a): The new density is

$$f_Y(y) = (1/2) f_X\left(\frac{(y-7)}{-2}\right).$$

The interval over which Y is distributed is clearly $[5, 7]$. Because of the reflection involved, the Y density would be a "backward ramp" over the interval $[5, 7]$. It reaches a max height of 1. This is enough info to draw the plot:



You compute the new mean and variance from the formulas:

$$\begin{aligned}\mu_Y &= (-2)\mu_X + 7 \\ \sigma_Y^2 &= (-2)^2\sigma_X^2 = 4\sigma_X^2\end{aligned}$$

part(b): From Appendix A, you see that a Uniform(0,1) RV has mean 1/2 and variance 1/12. You find a, b by solving the equations:

$$\begin{aligned}20 &= a(1/2) + b \\ 5 &= a^2(1/12)\end{aligned}$$

There are actually two solutions depending upon whether you take a to be positive or negative. Either answer is acceptable.

Solution to Problem 3.
part(a):

$$\phi_X(s) = E[e^{sX}] = (0.20)\exp(6s) + (0.16)\sum_{i=1}^{5}\exp(is).$$

part(b):

$$\phi_X'(s) = (1.2)\exp(6s) + (0.16)\sum_{i=1}^{5} i\exp(is).$$

Plugging in $s = 0$,

$$\mu_X = 1.2 + (0.16)[1 + 2 + 3 + 4 + 5] = 3.6.$$

part(c): Taking another derivative,

$$\phi_X''(s) = (7.2)\exp(6s) + (0.16)\sum_{i=1}^{5} i^2\exp(is).$$

Plugging in $s = 0$,

$$E[X^2] = 7.2 + (0.16)[1 + 4 + 9 + 16 + 25] = 16.$$

Therefore,

$$\sigma_X^2 = E[X^2] - \mu_X^2 = 16 - (3.6)^2 = 3.04.$$

Solution to Problem 4. (a): By inspection, the first cond PMF plot consists of 5 spikes of height 1/5 each at points 1,2,3,4,5. The second cond PMF plot consists of 5 spikes of height 1/5 at points 6,7,8,9,10.

(b): Let B_1, B_2 be the events that X takes a value to the left and right of 5.5, respectively. From the two individual cond PMF plots, it is clear that

$$\begin{aligned} P(3 \leq X \leq 7|B_1) &= 3/5 \\ P(3 \leq X \leq 7|B_2) &= 2/5 \end{aligned}$$

The overall prob should be

$$P(3 \leq X \leq 7) = P(B_1)(3/5) + P(B_2)(2/5) = (1/3)(3/5) + (2/3)(2/5) = 0.4667.$$

If we compute this prob directly from the original PMF, we get:

$$P^X(3) + P^X(4) + P^X(5) + P^X(6) + P^X(7) = 3(1/15) + 2(2/15) = 7/15 = 0.4667.$$

part(c): The two conditional means are the midpoints of their respective cond dists (by symmetry):

$$E(X|B_1) = 3, \quad E(X|B_2) = 8.$$

Therefore, the overall mean should be computable as

$$E(X) = (1/3)3 + (2/3)8 = 6.3333.$$

Computing $E(X)$ directly from the original PMF yields:

$$E(X) = (1/15)(1 + 2 + 3 + 4 + 5) + (2/15)(6 + 7 + 8 + 9 + 10) = 6.3333.$$

Solution to Problem 5. The original density is

$$f_X(x) = \begin{cases} Cx^2, & 1 \leq x \leq 5 \\ 0, & elsewhere \end{cases}$$

where $C = .0242$.

part(a): Let B_1, B_2 be the "left and right events". For the range from $x = 1$ to $x = 2.5$, we have

$$f_{X|B_1}(x) = \frac{Cx^2}{\int_1^{2.5} Cu^2 du} = (8/39)x^2 = (0.2051)x^2, \quad 1 \leq x \leq 2.5$$

(The cond density is zero elsewhere.) For the range $x = 2.5$ to $x = 5$, we have

$$f_{X|B_2}(x) = \frac{Cx^2}{\int_{2.5}^{5} Cu^2 du} = (24/875)x^2 = (0.0274)x^2, \quad 2.5 \leq x \leq 5$$

(The cond density is zero elsewhere.)

part(b):

$$E(X|B_1) = \int_1^{2.5} x(8/39)x^2 dx = 203/104 = 1.9519.$$

$$E(X|B_2) = \int_{2.5}^{5} x(24/875)x^2 dx = 225/56 = 4.0179.$$

The overall mean should be computable as:

$$E(X) = P(B_1)E(X|B_1) + P(B_2)E(X|B_2) = (117/992)1.9519 + (875/992)4.0179 = 3.7742.$$

Doing the computation directly, we get

$$E(X) = \int_1^5 Cx^3 dx = 117/31 = 3.7742.$$

part(c):

$$E(X^2|B_1) = \int_1^{2.5} x^2(8/39)x^2 dx = 3.9654.$$

$$E(X^2|B_2) = \int_{2.5}^{5} x^2(24/875)x^2 dx = 16.6071$$

The conditional variances are therefore:

$$Var(X|B_1) = 3.9654 - (1.9519)^2 = 0.1555$$

$$Var(X|B_2) = 16.6071 - (4.0179)^2 = 0.4636.$$

This gives us

$$Var(X|B_1)P(B_1) + Var(X|B_2)P(B_2) = 0.1555(117/992) + 0.4636(875/992) = 0.4273.$$

On the other hand,

$$Var(X) = \int_1^5 x^2(Cx^2)dx - (3.7742)^2 = 15.1161 - (3.7742)^2 = 0.8715.$$

Therefore

$$Var(X) \neq Var(X|B_1)P(B_1) + Var(X|B_2)P(B_2).$$