Solutions to EE 3025 Recitation 10 Review Problems for Exam 2

1. Let RV X have variance 100 and let RV Y have variance 400.

(a) If the correlation coefficient $\rho_{X,Y}$ is 1/2, compute $Var(X+Y)$ and $Var(X-Y)$.
Solution. We have

$$Cov(X,Y) = \rho\sigma_X\sigma_Y = (1/2)(10)(20) = 100.$$

Therefore,

$$\begin{aligned} Var(X+Y) &= Var(X) + Var(Y) + 2Cov(X,Y) = 700 \\ Var(X+Y) &= Var(X) + Var(Y) - 2Cov(X,Y) = 300 \end{aligned}$$

(b) On the other hand, if $Var(X+Y) = 400$ and $Var(X-Y) = 600$, figure out what the correlation coefficient $\rho_{X,Y}$ is. Then, compute $Var(3X-2Y)$.
Solution. We have

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 500 + 2Cov(X,Y) = 400$$

and therefore

$$Cov(X,Y) = -50.$$

(The $Var(X-Y)$ condition is not needed.) Then

$$\rho = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{-50}{(10)(20)} = -1/4.$$

$$Var(3X-2Y) = 9Var(X)+4Var(Y)-12Cov(X,Y) = 9*100+4*400-12*(-50) = 3100.$$

2. Let RV X have variance 100 and let RV Y have variance 400. Let $Cov(X,Y)$ be 100.

(a) Find the constant C that makes

$$Cov(X, X-CY) = 0.$$

In other words, you are making X and $X-CY$ uncorrelated.
Solution.

$$Cov(X, X-CY) = Cov(X,X) - C * Cov(X,Y) = 100 - C * 100 = 0.$$

Take $C = 1$.

(b) Find the constant D that makes

$$Var(X-DY)$$

a minimum.
Solution.

$$Var(X-DY) = Var(X) + D^2Var(Y) - 2D * Cov(X,Y) = 100 * 400D^2 - 200D.$$

Set the derivative equal to zero:

$$800D - 200 = 0.$$

You see that $D = 1/4$.

3. Discrete RV's X, Y are independent. The PDF of X is

$$(0.2)\delta(x-1) + (0.4)\delta(x-3) + (0.4)\delta(x-4).$$

The PDF is Y is

$$(0.4)\delta(y+1) + (0.6)\delta(y-2).$$

(a) Use convolution to find the PDF of $U = X + Y$.

Solution. Think of these as two functions of time that you are convoluting:

$$(0.2)\delta(t-1) + (0.4)\delta(t-3) + (0.4)\delta(t-4)$$

and

$$(0.4)\delta(t+1) + (0.6)\delta(t-2)$$

First, convolute the 1st signal with $(0.4)\delta(t+1)$:

$$0.4[(0.2)\delta(t) + (0.4)\delta(t-2) + (0.4)\delta(t-3)]$$

Then, convolute the 1st signal with $(0.6)\delta(t-2)$:

$$0.6[(0.2)\delta(t-3) + (0.4)\delta(t-5) + (0.4)\delta(t-6)]$$

Then add the two results, writing as a function of U:

$$0.08\delta(u) + 0.16\delta(u-2) + 0.28\delta(u-3) + 0.24\delta(u-5) + 0.24\delta(u-6)$$

(b) Use convolution to find the PDF of $V = 2X - Y$.

Solution. First, write down the PDF of $2X$ as a function of t:

$$(0.2)\delta(t-2) + (0.4)\delta(t-6) + (0.4)\delta(t-8).$$

Then, write down the PDF of $-Y$ as a function of t:

$$(0.4)\delta(t-1) + (0.6)\delta(t+2).$$

Convolute the 1st signal with $(0.4)\delta(t-1)$:

$$0.4[(0.2)\delta(t-3) + (0.4)\delta(t-7) + (0.4)\delta(t-9)]$$

Convolute the 1st signal with $(0.6)\delta(t+2)$:

$$0.6[(0.2)\delta(t) + (0.4)\delta(t-4) + (0.4)\delta(t-6)]$$

Add the two results, writing as a function of v:

$$0.12\delta(v) + 0.08\delta(v-3) + 0.24\delta(v-4) + 0.24\delta(v-6) + 0.16\delta(v-7) + 0.16\delta(v-9).$$

4. Let I, R be nonnegative continuously distributed RV's. Use the CDF method to show that $V = IR$ has PDF

$$f_V(v) = \int_0^\infty \frac{1}{i} f_I(i) f_R(v/i) di, \quad v \geq 0 \text{ (zero elsewhere)}.$$

Solution. For $v \geq 0$,

$$F_V(v) = P[V \leq v] = P[IR \leq v] = \int_0^\infty P[IR \leq v | I = i] f_I(i) di.$$

Now

$$P[IR \leq v | I = i] = P[iR \leq v | I = i] = P[R \leq v/i] = F_R(v/i).$$

Therefore,

$$F_V(v) = \int_0^\infty F_R(v/i) f_I(i) di.$$

Differentiating both sides,

$$f_V(v) = dF_V(v)/dv = \int_0^\infty \partial F_R(v/i) f_I(i)/\partial v di.$$

Plugging in

$$\partial F_R(v/i) f_I(i)/\partial v = (1/i) f_R(v/i) f_I(i),$$

we obtain the desired result.

5. X, Y have the bivariate Gaussian density

$$f_{X,Y}(x, y) = C \exp(-0.5[x^2 - 2xy + 4y^2]).$$

(a) What are $\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho_{X,Y}$?

Solution. Since there are no x, y terms in the exponent,

$$\mu_X = \mu_Y = 0.$$

Note that

$$x^2 - 2xy + 4y^2 = (x \;\; y) \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} (x \;\; y)^T.$$

The inverse of the "matrix in the middle" is the covariance matrix, which is

$$\begin{pmatrix} 4/3 & 1/3 \\ 1/3 & 1/3 \end{pmatrix} = \begin{pmatrix} \sigma_X^2 & \sigma_{X,Y} \\ \sigma_{X,Y} & \sigma_Y^2 \end{pmatrix}$$

We have

$$\sigma_X^2 = 4/3, \quad \sigma_Y^2 = 1/3, \quad \sigma_{X,Y} = 1/3.$$

$$\rho = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = \frac{1/3}{\sqrt{4/3}\sqrt{1/3}} = 1/2.$$

(b) What are $E[X|Y=1]$ and $E[Y|X=1]$?

Solution. For every y,

$$E[X|Y=y] = \mu_X + \rho(\sigma_X/\sigma_Y)(y-\mu_Y) = y.$$

Therefore,

$$E[X|Y=1] = 1.$$

For every x,

$$E[Y|X=x] = \mu_Y + \rho(\sigma_Y/\sigma_X)(x-\mu_X) = x/4.$$

Therefore

$$E[Y|X=1] = 1/4.$$

6. Let X be Uniform(0,1).

(a) Find the PDF of $Y = X^3$.

Solution. Y ranges from 0 to 1. For each fixed y in this range,

$$F_Y(y) = P(Y \le y) = P(X^3 \le y) = P(X \le y^{1/3}) = y^{1/3}.$$

Differentiating,

$$f_Y(y) = (1/3)y^{-2/3}, \quad 0 \le y \le 1 \ (zero\ elsewhere).$$

(b) Find the PDF of $Y = X(1-X)$.

Solution. First, plot the curve $y = x(1-x)$ from $x = 0$ to $x = 1$. You'll see that Y ranges from 0 to 1/4. For each y between 0 and 1/4, there are two x values such that $x(1-x) = y$, namely, the values

$$x_1(y) = \frac{1-\sqrt{1-4y}}{2}, \quad x_2(y) = \frac{1+\sqrt{1-4y}}{2}.$$

From the curve $y = x(1-x)$, you will see that the event $\{Y \le y\}$ can be broken down as follows:

$$\{Y \le y\} = \{X \le x_1(y)\} \cup \{X \ge x_2(y)\}.$$

Therefore,

$$P(Y \le y) = P(X \le x_1(y)) + P(X \ge x_2(y)).$$

By symmetry, the two prob on the right side are equal and so

$$P(Y \le y) = 2P(X \le x_1(y)) = 2x_1(y) = 1 - \sqrt{1-4y}.$$

Differentiating,

$$f_Y(y) = \frac{2}{\sqrt{1-4y}}, \quad 0 \le y \le 1/4 \ (zero\ elsewhere).$$

7. Let X_1, X_2 be independent discrete RV's each taking the values $1, 2, 3$ with probability $1/3$ each.

(a) Find the PMF of $U = \max(X_1, X_2)$.

(b) Find the PMF of $V = \min(X_1, X_2)$.

Solution. There's a clever way to work (a),(b) in a unified way. Map (X_1, X_2) values into (U, V) values:

```
X1  X2  U  V
1   1   1  1
1   2   2  1
1   3   3  1
2   1   2  1
2   2   2  2
2   3   3  2
3   1   3  1
3   2   3  2
3   3   3  3
```

Each (X_1, X_2) value has prob $1/9$. This allows you to construct the following U, V joint prob table:

$$\begin{array}{c} \\ U=1 \\ U=2 \\ U=3 \end{array} \begin{array}{c} \begin{array}{ccc} V=1 & V=2 & V=3 \end{array} \\ \begin{pmatrix} 1/9 & 0 & 0 \\ 2/9 & 1/9 & 0 \\ 2/9 & 2/9 & 1/9 \end{pmatrix} \end{array}$$

The row sums and column sums are the PMF's of U, V, respectively.

8. Let X, Y be discrete RV's whose joint PMF takes the form

$$P[X = i, Y = j] = C|i - 2j|,$$

for $i, j = 1, 2, 3$ and 0 elsewhere.

(a) Find the constant C.

Solution. C must be the reciprocal of the sum

$$\sum_{i,j=1}^{3} |i - 2j|.$$

(b) Find the marginal PMF $p_X(x)$.

Solution.

$$p_X(i) = C \sum_{j=1}^{3} |i - 2j|, \quad i = 1, 2, 3.$$

(c) Find the conditional PMF of Y given $X = 2$.

Solution.

$$p_{Y|X}(y|x = 2) = \frac{C|2 - 2y|}{p_X(2)}, \quad y = 1, 2, 3.$$

(d) Compute $E[Y|X = 2]$.

Solution.

$$E[Y|X = 2] = \sum_{y=1}^{3} y[C|2 - 2y|/p_X(2)].$$

(e) Compute the correlation $E[XY]$.

Solution.

$$E[XY] = \sum_{i,j=1}^{3} Cij|i - 2j|.$$

9. Let X, Y be jointly continuous RV's whose joint PDF takes the form

$$f(x, y) = C(x + 2y),$$

for $0 \leq x \leq 2$ and $0 \leq y \leq 1$.

(a) Find the constant C.

Solution. C must be the reciprocal of the double integral

$$\int_0^1 \int_0^2 (x + 2y)dxdy.$$

(b) Find the marginal PDF $f_X(x)$.

Solution.

$$f_X(x) = \int_0^1 C(x + 2y)dy,$$

for $0 \leq x \leq 2$.

(c) Find the conditional PDF of Y given $X = 1$.

Solution.

$$f_{Y|X}(y|x = 1) = \frac{C(1 + 2y)}{f_X(1)}, \quad 0 \leq y \leq 1.$$

(d) Compute $P[Y < 1/2|X = 1]$.

Solution.

$$P(Y < 1/2|X = 1) = \int_0^{1/2} \frac{C(1 + 2y)}{f_X(1)}dy.$$

(e) Compute the correlation $E[XY]$.

Solution.

$$E[XY] = \int_0^1 \int_0^2 Cxy(x + 2y)dxdy.$$

10. Let (X, Y) be uniformly distributed over the triangular region inside the triangle in the xy-plane whose three vertices are $(0, 0), (5, 0), (3, 3)$

(a) Find μ_X, μ_Y.

Solution.

$$(\mu_X, \mu_Y) = (1/3)[(0, 0) + (5, 0) + (3, 3)].$$

(b) Find $E[X|Y=y]$ as a function of y. This is a straight line function of y. What is this straight line in terms of the triangular region? Indicate on a plot.

Solution. It's the median line that connects vertex $(3,3)$ to the midpoint of the opposite side.

11. Let X, Y be jointly continuously distributed with joint density

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le y \le x \le 1 \\ 0, & elsewhere \end{cases}$$

(a) Set up the double integral for $E[XY]$ with all 4 limits properly inserted. Do not integrate.

Solution.

$$E[XY] = \int_0^1 \int_0^x xyf(x,y)dydx.$$

(b) Set up $E[X|Y=0.5]$ as the ratio of two one-dimensional integrals with all the limits properly inserted. Do not integrate.

Solution.

$$E[X|Y=0.5] = \frac{\int_{1/2}^1 xf(x,y)dx}{\int_{1/2}^1 f(x,y)dx},$$

in which you substitute $y = 0.5$.

(c) Set up $P[X^2 + Y^2 \le 0.25]$ as a double integral in polar coordinates with all the limits properly inserted. Do not integrate.

Solution.

$$\int_0^{\pi/4} \int_0^{1/2} 2rdrd\theta.$$

12. Bill eats X ice cream cones, where X is Poisson with parameter 2. Bill then runs Y miles, where Y is the number of heads obtains in flipping a fair coin $X+2$ times. Use the law of iterated expectation to compute:

(a) $E[Y]$.

Solution.

$$E[Y|X] = (X+2)/2.$$

$$E[Y] = E[E[Y|X]] = (1/2)(E[X]+2) = 2.$$

(b) $Var[Y]$.

Solution.

$$Var[Y|X] = (X+2)/4.$$

$$E[Y^2|X] = E[Y|X]^2 + Var[Y|X] = (X+2)^2/4 + (X+2)/4.$$

$$E[Y^2] = E[(X+2)^2]/4 + E[X+2]/4.$$

Expand this using $E[X] = 2$, $E[X^2] = 6$. Then use the formula

$$Var[Y] = E[Y^2] - \mu_Y^2.$$

(c) $E[XY]$.

Solution.

$$E[XY|X] = XE[Y|X] = X(X+2)/2.$$
$$E[XY] = E[X(X+2)]/2.$$

(d) $Cov[X, Y]$.

Solution. It's $E[XY] - \mu_X \mu_Y$.

(e) $\rho_{X,Y}$.

Solution. It's $Cov[X, Y]$ divided by $\sigma_X \sigma_Y$. Use $\sigma_X = \sqrt{2}$.

13. Discrete random variables N and K have the joint PMF

$$P_{N,K}(n, k) = \begin{cases} \frac{100^n e^{-100}}{(n+1)!}, & k = 0, 1, \cdots, n;\ n = 0, 1, 2, \cdots \\ 0, & elsewhere \end{cases}$$

(a) Find $P_N(n)$.

Solution.

$$P_N(n) = \sum_{k=0}^{n} P_{N,K}(n, k) = (100)^n \exp(-100)/n!,\ n = 0, 1, 2, 3, \cdots.$$

This is the Poisson(100) distribution.

(b) Find $P_{K|N}(k|n)$.

Solution.

$$P_{K|N}(k|n) = \frac{P_{N,K}(n, k)}{P_N(n)} = 1/(n+1),\ k = 0, 1, \cdots, n.$$

In other words, this cond dist is DiscreteUniform$(0, n)$.

(c) Compute $E[K|N = n]$.

Solution. $E[K|N = n] = (0 + n)/2$, the mean of DiscreteUniform$(0, n)$. (See Appendix A.)

(d) Compute $E[K]$ via law of iterated expectation formula

$$E[K] = E[E[K|N]].$$

Solution. $E[K|N] = N/2$. Therefore,

$$E[K] = E[E[K|N]] = E[N/2] = 50,$$

since N is Poisson with mean 100.

14. $X_1, X_2, \cdots, X_{10}$ are independent RV's which are each uniformly distributed bewteeen -2 and 2. Let

$$S = X_1 + X_2 + \cdots + X_{10}.$$

(a) Determine the correlation between S and the RV

$$U = X_1 - X_2 + X_3 - X_4 + X_5 - X_6 + X_7 - X_8 + X_9 - X_{10}$$

(b) Determine the correlation between S and the RV

$$V = X_1 - X_2 + X_3 - X_4 + X_5$$

(c) Determine the correlation between S and the RV

$$W = X_6 + X_7 + X_8 + X_9 + X_{10}$$

Solution. For $i \neq j$,

$$E[X_i X_j] = E[X_i]E[X_j] = 0 * 0 = 0.$$

Therefore, if you have to compute the expected value of the product of two linear comb's of the X_i's, you only have to pay attention to product terms of the form $X_i * X_i = X_i^2$.

Solution to (a). The only X_i^2 terms appearing in the product of S times U are:

$$X_1^2 - X_2^2 + X_3^2 - X_4^2 + X_5^2 - X_6^2 + X_7^2 - X_8^2 + X_9^2 - X_{10}^2$$

When you take the expected value term by term, the expected values can cancel out:

$$EX_1^2] - E[X_2^2] + \cdots + E[X_9^2] - E[X_{10}^2] = 0.$$

Solution to (b). Just paying attention to the expected values of the X_i^2 terms in SV:

$$= E[X_1^2] - E[X_2^2] + E[X_3^2] - E[X_4^2] + E[X_5^2] = E[X_5^2] = Var(X_5) = 4^2/12 = 4/3$$

Solution to (c). By inspection, you'll get 5 times $E[X_1]^2$, which is $20/3$.

15. X and Y are jointly Gaussian RV's with $E[X] = E[Y] = 0$ and $Var[X] = Var[Y] = 1$. Furthermore,

$$E[Y|X] = X/2.$$

What is the joint PDF of X and Y?

Solution. We have

$$\mu_{Y|X=x} = x/2.$$

The coefficient of x, which is $1/2$, must be equal to

$$1/2 = \rho\sigma_Y/\sigma_X = \rho.$$

Therefore,

$$Var(Y|X) = \sigma_Y^2(1-\rho^2) = 3/4.$$

The conditional density of Y given $X = x$ is therefore

$$\frac{1}{\sqrt{2\pi}\sqrt{3/4}}\exp(-0.5\frac{(y-\mu_{Y|X=x})^2}{3/4}).$$

Multiplying this by the Gaussian(μ_X,σ_X^2) density, we get the joint density

$$\frac{1}{\sqrt{2\pi}}\exp(-0.5x^2)\frac{1}{\sqrt{2\pi}\sqrt{3/4}}\exp(-2(y-x/2)^2/3),$$

which simplifies to

$$\frac{1}{2\pi\sqrt{3/4}}\exp(-2\{x^2+y^2-xy\}/3).$$

16. Three RV's X, Y, Z have a "trivariate" Gaussian distribution in which their joint density $f(x, y, z)$ takes the form

$$f(x,y,z) = C\exp\left(-0.5[3x^2+6y^2+4z^2+4xy-2xz+2yz]\right),$$

where C is a (unique) positive constant that one does not need to know in order to work this problem.

(a) Compute the three variances σ_X^2, σ_Y^2, σ_Z^2. (Hint: Invert a certain 3×3 matrix.)

Solution. You get the covariance matrix by inverting the matrix

```
 3  2  -1
 2  6   1
-1  1   4
```

which gives

```
 23/43        -9/43         8/43
 -9/43        11/43        -5/43
  8/43        -5/43        14/43
```

Therefore,

$$\sigma_X^2 = 23/43,\quad \sigma_Y^2 = 11/43,\quad \sigma_Z^2 = 14/43.$$

(b) Compute the three correlation coefficients $\rho_{X,Y}$, $\rho_{X,Z}$, $\rho_{Y,Z}$.

Solution.

$$\begin{aligned}\rho_{X,Y} &= \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{-9/43}{\sigma_X\sigma_Y} = -0.5658\\ \rho_{X,Z} &= \frac{Cov(X,Z)}{\sigma_X\sigma_Z} = \frac{8/43}{\sigma_X\sigma_Z} = -0.4458\\ \rho_{Y,Z} &= \frac{Cov(Y,Z)}{\sigma_Y\sigma_Z} = \frac{-5/43}{\sigma_Y\sigma_Z} = -0.4029\end{aligned}$$

(c) Determine the joint density of Y and Z. (Hint: This is a *bivariate* Gaussian density.)

Solution. The means of X, Y, Z are each zero (no linear terms in the exponent of the trivariate density). The means of Y and Z are therefore zero, and the covariance matrix of Y and Z is

```
11/43  -5/43
-5/43  14/43
```

The inverse of this covariance matrx is

```
14/3   5/3
 5/3  11/3
```

The Y, Z joint density is therefore of the form

$$C \exp(-0.5\{14y^2/3 + 10yz/3 + 11z^2/3\}),$$

where

$$C = \frac{1}{2\pi\sqrt{1-\rho^2_{Y,Z}}} = 0.1739.$$

17. Let random variables X, Y be jointly continuously distributed in the square

$$\{(x, y) : 0 \leq x \leq 3; 0 \leq y \leq 3\}.$$

Suppose their joint CDF satisfies

$$\begin{aligned}
F_{X,Y}(1,1) &= 1/81 \\
F_{X,Y}(2,1) &= 4/81 \\
F_{X,Y}(3,1) &= 1/9 \\
F_{X,Y}(1,2) &= 4/81 \\
F_{X,Y}(2,2) &= 16/81 \\
F_{X,Y}(3,2) &= 4/9 \\
F_{X,Y}(1,3) &= 1/9 \\
F_{X,Y}(2,3) &= 4/9
\end{aligned}$$

Compute

$$P[(X, Y) \in R],$$

where R is the cross-shaped region in the first quadrant of the xy-plane given by

$$R = \{(x, y) : 0 \leq x \leq 3;\ \ 1 \leq y \leq 2\} \cup \{(x, y) : 1 \leq x \leq 2;\ \ 0 \leq y \leq 3\}.$$

(Hint: Partition R into 3 rectangular pieces and find the prob (X, Y) belongs to each piece.)

Solution. Abbreviate $F_{X,Y}(x, y)$ as $F(x, y)$. For the rectangle (square) with four corners

$$(1, 1), (1, 2), (0, 1), (0, 2),$$

the prob (X, Y) falls inside is

$$F(1, 2) + F(0, 1) - F(1, 1) - F(0, 2) = 3/81.$$

For the rectangle with four corners

$$(2,0),(2,3),(1,3),(1,0),$$

the prob (X,Y) falls inside is

$$F(2,3)+F(1,0)-F(2,0)-F(1,3)=1/3.$$

For the rectangle (square) with four corners

$$(3,1),(3,2),(2,2),(2,1),$$

the prob (X,Y) falls inside is

$$F(3,2)+F(2,1)-F(3,1)-F(2,2)=15/81.$$

Therefore, the prob (X,Y) falls in the union of these 3 rectangles is

$$(3/81)+(1/3)+(15/81)=5/9.$$

18. Problem 5.6.7, page 241, of your textbook.

Solution. The pairs (Y_1,Y_2) and (Y_3,Y_4) have the same probability distribution; they are also independent pairs. Therefore the mean vector of (Y_1,Y_2,Y_3,Y_4) will have the form

```
a b a b
```

where a,b are the means of Y_1,Y_2, respectively, and the covariance matrix of (Y_1,Y_2,Y_3,Y_4) will have the form

```
c d 0 0
e f 0 0
0 0 c d
0 0 e f
```

where

```
c d
e f
```

is the covariance matrix of (Y_1,Y_2).

By factorization, (Y_1,Y_2) has joint PDF equal to 2 over the region

$$\{0\le y_1\le y_2\le 1\},$$

and so

$$
\begin{aligned}
E[Y_1] &= \int_0^1 \int_{y_1}^1 y_1(2)dy_2dy_1 = 1/3 \\
E[Y_2] &= \int_0^1 \int_{y_1}^1 y_2(2)dy_2dy_1 = 2/3 \\
E[Y_1Y_2] &= \int_0^1 \int_{y_1}^1 y_1y_2(2)dy_2dy_1 = 1/4 \\
Cov(Y_1, Y_2) &= 1/4 - (1/3)(2/3) = 1/36 \\
E[Y_1^2] &= \int_0^1 \int_{y_1}^1 y_1^2(2)dy_2dy_1 = 1/6 \\
E[Y_2^2] &= \int_0^1 \int_{y_1}^1 y_2^2(2)dy_2dy_1 = 1/2 \\
Var(Y_1) &= 1/6 - (1/3)^2 = 1/18 \\
Var(Y_2) &= 1/2 - (2/3)^2 = 1/18
\end{aligned}
$$

The mean vector of (Y_1, Y_2, Y_3, Y_4) is therefore

$$(1/3, 2/3, 1/3, 2/3)^T,$$

and the covariance matrix is

```
1/18   1/36   0      0
1/36   1/18   0      0
 0      0    1/18   1/36
 0      0    1/36   1/18
```

We used the equation

$$R_Y = C_Y + \mu_Y \mu_Y^T$$

to compute the correlation matrix of (Y_1, Y_2, Y_3, Y_4):

```
    1/6           1/4           1/9           2/9
    1/4           1/2           2/9           4/9
    1/9           2/9           1/6           1/4
    2/9           4/9           1/4           1/2
```