

8 Rec 8: Miscellaneous End of Chapter 4 Topics

Directions: Your instructor will spend the the first 40 minutes of the recitation period working some review problems and going over one or more Matlab experiments in the following. During the last 10 minutes of recitation, your proctor will give you a “Lab Form” that your recitation team completes, signs, and turns in. See the last page for an indication of what you will be asked to do on the Lab Form.

Due to time limitations, only a part of the following can be covered during the recitation period. However, you might want in the future to try some of the uncovered experiments on your own. They could give skills useful on some future homework problems and could lend insight into your understanding of the course from an experimental point of view.

This Week’s Topics. This week’s recitation gives you insight into the remaining topics from Chapter 4. Since these remaining topics are not all that related to one another, this week’s recitation may seem like somewhat of a “potpourri”. Here are the topics to be covered:

- Correlation Matrices of Linearly Transformed RV’s
- Whitening Filter Design
- Conditional PDF’s of Joint Gaussian Density
- Expected Lifetime of Relay Circuits

8.1 Exp 1: Correlation Matrices of Linearly Transformed RV’s

In Experiment 5 of Recitation 7, the concepts of correlation matrix and covariance matrix of a set of RV’s were introduced. Suppose we now linearly transform a set of RV’s to obtain a new set of RV’s. This experiment shows you how to obtain the correlation matrix and covariance matrix of the new set of RV’s from the original correlation and covariance matrices.

Let X_1, X_2 be given RV’s and let Y_1, Y_2 be RV’s obtained from X_1, X_2 via a linear transformation. We can express the relationship between Y_1, Y_2 and X_1, X_2 using matrices:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (1)$$

How can the correlation $E[Y_1 Y_2]$ between Y_1, Y_2 be computed from the correlation $E[X_1 X_2]$ between X_1, X_2 ? Here is a matrix method for doing this:

- Let A be the 2×2 coefficient matrix in (1):

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}.$$

Then

$$\begin{bmatrix} E[Y_1^2] & E[Y_1Y_2] \\ E[Y_1Y_2] & E[Y_2^2] \end{bmatrix} = A \begin{bmatrix} E[X_1^2] & E[X_1X_2] \\ E[X_1X_2] & E[X_2^2] \end{bmatrix} A^T \quad (2)$$

- *Example 1.* Let (X_1, X_2) be independent standard Gaussian random variables. Let (Y_1, Y_2) be the jointly Gaussian random variables obtained as follows:

$$\begin{aligned} Y_1 &= 5X_1 + 3X_2 \\ Y_2 &= 4X_1 - 2X_2 \end{aligned}$$

Use Matlab to compute the correlation $E[Y_1Y_2]$ by taking a product of three 2×2 matrices according to equation (2) (the middle one of which is the 2×2 identity matrix) and then picking off the desired correlation as a certain element of the 2×2 product matrix.

- *Example 2.* Now let X_1, X_2 be independent RV's each uniformly distributed between 0 and 1. Use the same linear transformation as in Example 1 to obtain correlated RV's Y_1, Y_2 . Use Matlab to compute the correlation $E[Y_1Y_2]$ by taking a product of three 2×2 matrices according to equation (2). Be careful: The middle matrix in (2) is no longer the identity matrix—the two diagonal elements are each equal to the second moment of the uniform $[0, 1]$ distribution, which is $1/3$, and the two off diagonal elements are both $1/4$ (why?).

We can easily extend these ideas to linear transformations of three or more RV's. If we have

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

then

$$\begin{bmatrix} E[Y_1^2] & E[Y_1Y_2] & E[Y_1Y_3] \\ E[Y_1Y_2] & E[Y_2^2] & E[Y_2Y_3] \\ E[Y_1Y_3] & E[Y_2Y_3] & E[Y_3^2] \end{bmatrix} = A \begin{bmatrix} E[X_1^2] & E[X_1X_2] & E[X_1X_3] \\ E[X_1X_2] & E[X_2^2] & E[X_2X_3] \\ E[X_1X_3] & E[X_2X_3] & E[X_3^2] \end{bmatrix} A^T \quad (3)$$

where the matrix A is now the following 3×3 coefficient matrix:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}.$$

- *Example 3.* Let X_1, X_2, X_3 be independent standard Gaussian random variables and let Y_1, Y_2, Y_3 be the random variables:

$$\begin{aligned} Y_1 &= 4X_1 - 9X_2 + 5X_3 \\ Y_2 &= -2X_1 + 3X_2 + 7X_3 \\ Y_3 &= 3X_1 - 5X_2 + X_3 \end{aligned}$$

Use Matlab to compute the three correlations $E[Y_1Y_2]$, $E[Y_1Y_3]$, $E[Y_2Y_3]$ by taking a product of three 3×3 matrices according to equation (3) (the middle one of which is the 3×3 identity matrix) and then picking off the desired correlations as certain elements of the 3×3 product matrix.

- *Example 4.* Now let X_1, X_2, X_3 be independent RV's each uniformly distributed between 0 and 1. Use the same linear transformation as in previous Example 3 to obtain correlated RV's Y_1, Y_2, Y_3 . Use Matlab to compute the three correlations $E[Y_1Y_2]$, $E[Y_1Y_3]$, $E[Y_2Y_3]$ by taking a product of three 3×3 matrices according to equation (3). Be careful: The middle matrix in (3) is no longer the identity matrix.

Now suppose we introduce a bias term $[b_1 \ b_2]^T$ in (1):

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

The means of the new RV's are related to the means of the old RV's in the same way:

$$\begin{bmatrix} \mu_{Y_1} \\ \mu_{Y_2} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} \mu_{X_1} \\ \mu_{X_2} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

The covariance and variance of the new RV's are related to the covariance and variance of the old random variables as follows:

$$\begin{bmatrix} \sigma_{Y_1}^2 & \sigma_{Y_1Y_2} \\ \sigma_{Y_1Y_2} & \sigma_{Y_2}^2 \end{bmatrix} = A \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1X_2} \\ \sigma_{X_1X_2} & \sigma_{X_2}^2 \end{bmatrix} A^T$$

Notice that the bias term did not affect these covariance and variance computations at all (you get the same answers taking the bias term $[b_1 \ b_2]^T$ to be zero).

- *Example 5.* Let U, V be independent standard Gaussian RV's. Let X, Y be the dependent Gaussian RV's defined by

$$\begin{aligned} X &= 5U + 3V - 7 \\ Y &= 4U - 2V + 3 \end{aligned}$$

Do the following Matlab computations to compute $\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho_{X,Y}$. First, compute the means μ_X, μ_Y by executing the following Matlab code:

```
mu_U=0;
mu_V=0;
A = [5 3
      4 -2];
b = [-7 3];
newmean = A*[mu_U mu_V]' + b';
mu_X = newmean(1)
mu_Y = newmean(2)
```

Now we use the equation

$$\begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix} = A \begin{bmatrix} \sigma_U^2 & \sigma_{UV} \\ \sigma_{UV} & \sigma_V^2 \end{bmatrix} A^T \quad (4)$$

to compute $\sigma_X, \sigma_Y, \rho_{X,Y}$ by executing:

```

C = A*A';
sigma_X = sqrt(C(1,1))
sigma_Y = sqrt(C(2,2))
rho_XY = C(1,2)/(sigma_X*sigma_Y)

```

(We used the fact that the middle matrix in the triple product on right side of (4) is the 2×2 identity matrix.)

- *Example 6.* Rework Example 5 now assuming that U, V are independent uniform $[0, 1]$ RV's. Compute $\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho_{X,Y}$ via Matlab. (Note: The middle term on the right side of equation (4) is no longer an identity matrix, but it is a diagonal matrix—what is it?)

We conclude by extending these ideas to linear transformations of three RV's with a bias term:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The new covariances are expressible in terms of the old covariances via the matrix equation:

$$\begin{bmatrix} \sigma_{Y_1}^2 & \sigma_{Y_1 Y_2} & \sigma_{Y_1 Y_3} \\ \sigma_{Y_1 Y_2} & \sigma_{Y_2}^2 & \sigma_{Y_2 Y_3} \\ \sigma_{Y_1 Y_3} & \sigma_{Y_2 Y_3} & \sigma_{Y_3}^2 \end{bmatrix} = A \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1 X_2} & \sigma_{X_1 X_3} \\ \sigma_{X_1 X_2} & \sigma_{X_2}^2 & \sigma_{X_2 X_3} \\ \sigma_{X_1 X_3} & \sigma_{X_2 X_3} & \sigma_{X_3}^2 \end{bmatrix} A^T \quad (5)$$

where the matrix A is now the following 3×3 coefficient matrix:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}.$$

The bias term does not affect the covariance computations, but it does affect the computations of the correlations $E[Y_i Y_j]$. The simplest way to compute the correlations would be to first compute the covariances and then to compute the correlations from the covariances, as follows:

$$\begin{bmatrix} E[Y_1^2] & E[Y_1 Y_2] & E[Y_1 Y_3] \\ E[Y_1 Y_2] & E[Y_2^2] & E[Y_2 Y_3] \\ E[Y_1 Y_3] & E[Y_2 Y_3] & E[Y_3^2] \end{bmatrix} = \begin{bmatrix} \sigma_{Y_1}^2 & \sigma_{Y_1 Y_2} & \sigma_{Y_1 Y_3} \\ \sigma_{Y_1 Y_2} & \sigma_{Y_2}^2 & \sigma_{Y_2 Y_3} \\ \sigma_{Y_1 Y_3} & \sigma_{Y_2 Y_3} & \sigma_{Y_3}^2 \end{bmatrix} + M * M^T, \quad (6)$$

where M is the column vector of means computable as

$$M = \begin{bmatrix} \mu_{Y_1} \\ \mu_{Y_2} \\ \mu_{Y_3} \end{bmatrix} = A * \begin{bmatrix} \mu_{X_1} \\ \mu_{X_2} \\ \mu_{X_3} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- *Example 7.* Let X_1, X_2, X_3 be independent uniform $[0, 1]$ random variables and let Y_1, Y_2, Y_3 be the random variables:

$$\begin{aligned} Y_1 &= 4X_1 - 9X_2 + 5X_3 - 2 \\ Y_2 &= -2X_1 + 3X_2 + 7X_3 + 4 \\ Y_3 &= 3X_1 - 5X_2 + X_3 + 3 \end{aligned}$$

Use Matlab to find the correlation matrix

$$\begin{bmatrix} E[Y_1^2] & E[Y_1Y_2] & E[Y_1Y_3] \\ E[Y_1Y_2] & E[Y_2^2] & E[Y_2Y_3] \\ E[Y_1Y_3] & E[Y_2Y_3] & E[Y_3^2] \end{bmatrix}$$

according to the method just discussed (i.e., find the covariance matrix using (5) and then find the correlation matrix using (6)).

8.2 Exp 2: Whitening Filter Design

In this experiment, you are going to design a type of *whitening filter*. Here is a block diagram illustrating what you will be attempting to do:



As inputs to the whitening filter, you will be given two data streams \mathbf{u} , \mathbf{v} , each consisting of 50000 samples. The outputs from the whitening filter are to be two data streams \mathbf{x} , \mathbf{y} , also each consisting of 50000 samples. The inputs and outputs are related by the equations

$$\begin{aligned} \mathbf{x} &= \mathbf{a}\mathbf{u} + \mathbf{b}\mathbf{v}; \\ \mathbf{y} &= \mathbf{c}\mathbf{u} + \mathbf{d}\mathbf{v}; \end{aligned}$$

where the 4 real parameters

$$a, b, c, d \tag{7}$$

are called the *whitening filter coefficients*. In this experiment, you will be designing the whitening filter coefficients so that the two output streams will be orthogonal, that is, so that

$$\text{mean}(\mathbf{x} \cdot \mathbf{y}) = 0$$

There are several approaches via which the whitening filter coefficients (7) can be designed. The approach we show you here uses the eigendecomposition of the 2×2 correlation matrix of the data vectors \mathbf{u}, \mathbf{v} (which is obtained with the Matlab function `eig`). We will ultimately present some theory in one of the lectures justifying this approach; however, you do not need to understand the theory in order to perform the method. You just follow the following steps.

Step 1: Run the following lines of Matlab code:

```
M=randn(2,50000);
u=sum(abs(M)); v=sqrt(sum(M.^2));
```

This will store the streams \mathbf{u}, \mathbf{v} , that you will be using in Matlab memory.

Step 2: Compute the 2×2 correlation matrix of the data streams \mathbf{u}, \mathbf{v} , by running the following single line of Matlab code:

```
CORRMATRIX = [u;v]*[u;v]'/50000
```

It is fun to see what the elements of this “sample” correlation matrix are estimating. Let S, T be independent standard Gaussian RV’s. Let U, V be the RV’s

$$\begin{aligned} U &= |S| + |T| \\ V &= \sqrt{S^2 + T^2} \end{aligned}$$

Then the entries of vector \mathbf{u} can be regarded as samples of RV U and the entries of vector \mathbf{v} can be regarded as samples of RV V . It follows that the sample correlation matrix “CORRMATRIX” is estimating the “theoretical correlation matrix”

$$\begin{bmatrix} E[U^2] & E[UV] \\ E[UV] & E[V^2] \end{bmatrix}$$

Matlab can help you find this theoretical correlation matrix. For example,

$$E[U^2] = E[(|S|+|T|)^2] = E[S^2]+E[T^2]+2E[|S|]E[|T|] = 2+2 \left(\frac{2}{\sqrt{2\pi}} \int_0^\infty s \exp(-s^2/2) ds \right)^2.$$

You can now use the Matlab function “`int`” as an aid in evaluating $E[U^2]$. What do you get? Similarly,

$$E[V^2] = E[S^2 + T^2] = 2.$$

You should now check whether the theoretical values of the second moments $E[U^2]$ and $E[V^2]$ correspond to the diagonal entries of CORRMATRIX. One can also compute the correlation $E[UV]$ in terms of S and T as

$$E[UV] = E[(|S| + |T|)\sqrt{S^2 + T^2}].$$

The right hand side can be evaluated by a conversion to polar coordinates. Your instructor might show you some of this computation on the board.

Step 3: Compute the following 2×2 matrix A by running the line of code:

```
[A,B] = eig(CORRMATRIX); A
```

The columns of the matrix A are linearly independent eigenvectors of $CORRMATRIX$. Let the whitening filter parameters a, b, c, d be selected as follows:

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}.$$

Step 4: In this step, you pass the data vectors u, v through the whitening filter and check whether the filter has whitened the data:

```
x = a*u+b*v  
y = c*u+d*v;  
mean(x.*y)
```

Did you get the correlation figure

```
mean(x.*y)
```

to be zero?

Step 5: This step will give you some additional insight into why the whitening filter worked. You can regard the filter output vectors x and y as samples of RV's X and Y , respectively, where X, Y are obtained as linear combinations of U, V which we can write in the following matrix format:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = A^T * \begin{bmatrix} U \\ V \end{bmatrix}$$

As a consequence of this transformation, you can use what you learned in Experiment 1 to conclude that

$$\begin{bmatrix} E[X^2] & E[XY] \\ E[XY] & E[Y^2] \end{bmatrix} = A^T * \begin{bmatrix} E[U^2] & E[UV] \\ E[UV] & E[V^2] \end{bmatrix} * A. \quad (8)$$

It is interesting to see what the 2×2 matrix on the left turns out to be. You can obtain some insight into this using Matlab as follows: Do the following matrix triple product via Matlab

```
A'*CORRMATRIX*A
```

Do you obtain a diagonal matrix? Interpret what this means in terms of the left side of equation (8). Does it now make more sense why the x data and the y data have 0 correlation? If you are totally confused at this point, your instructor can try to provide more explanation.

8.3 Exp 3: Conditional PDF's of Joint Gaussian Density

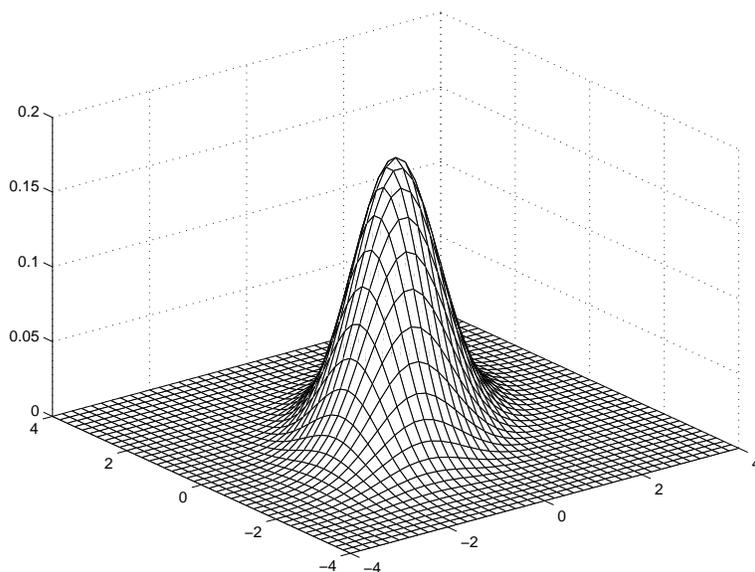
In Experiment 5 of Recitation 6, we introduced you to the joint Gaussian density surface $z = f(x, y)$, where $f(x, y)$ is the joint Gaussian density function. Again, take $f(x, y)$ of the form

$$f(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right), \quad (9)$$

meaning that we have a joint Gaussian pair of RV's (X, Y) with means μ_X and μ_Y equal to zero, standard deviations σ_X and σ_Y equal to one, and correlation coefficient ρ strictly between -1 and 1. In this experiment, we show you how vertical cross-sections of the Gaussian density surface are related to conditional PDF's of the joint Gaussian RV's X, Y . To obtain a vertical cross-section, we will cut through the density surface with planes perpendicular to the xy -plane. There are two types of vertical cross-sections:

- Cut through the surface with a plane of form $x = C$, where C is a constant. Up to a scaling factor, this will yield the conditional density $f_{Y|X}(y|x = C)$ of Y given $X = C$, which is a Gaussian density.
- Cut through the surface with a plane of form $y = C$, where C is a constant. Up to a scaling factor, this will yield the conditional density $f_{X|Y}(x|y = C)$ of X given $Y = C$, a Gaussian density.

Example 8. Throughout this example, we assume (X, Y) to have correlation coefficient $\rho = 1/2$. Using the Matlab script in Experiment 5 of Recitation 6, you can easily obtain the surface plot $z = f(x, y)$, which is given by the following figure:

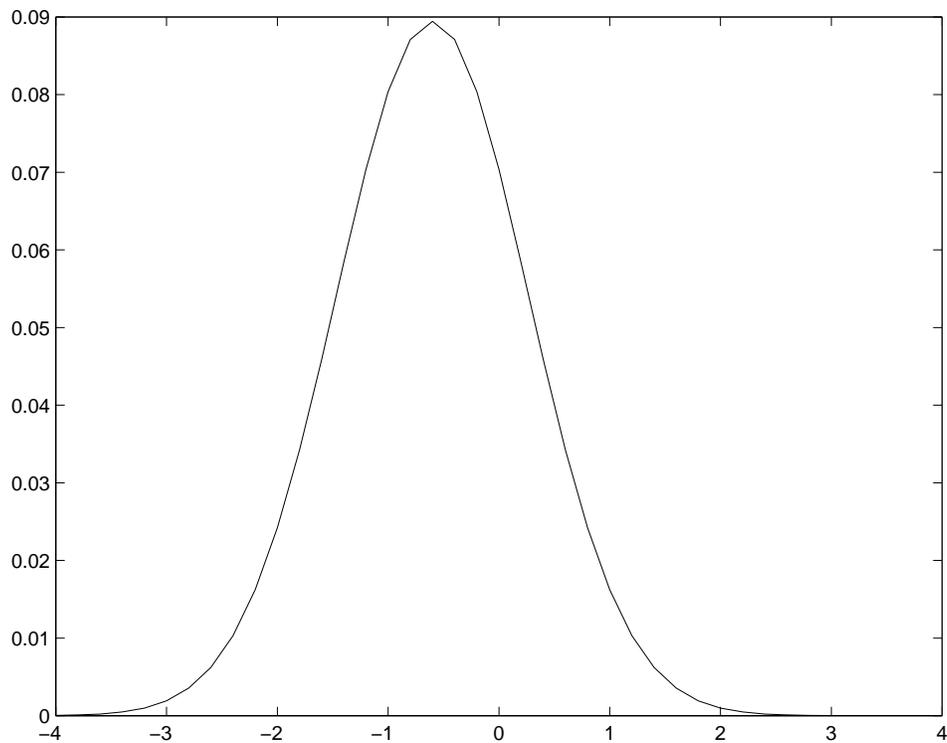


The contours that you see on the surface are the cross-sections of the density surface that you obtain with planes of the form $x = C$ and $y = C$, where C is a constant.

- We run the following code, which generates the plot of one of these contours:

```
x=-4:.2:4;
y=-4:.2:4;
[X,Y]=meshgrid(x,y);
rho=1/2;
Z=1/(2*pi*sqrt(1-rho^2))*exp(-(X.^2-2*rho*X.*Y + Y.^2)/(2*(1-rho^2)));
z1=Z(:,15);
plot(y,z1);
```

Your plot should look like this:



- Notice that the plot has the shape of a Gaussian density curve. Let's investigate further. In going down columns of Z , you are fixing the x coordinate and letting the y coordinate vary. For example, if there are 4 values x_1, x_2, x_3, x_4 in vector \mathbf{x} and 4 values y_1, y_2, y_3, y_4 in vector \mathbf{y} , then the matrix Z would take the form

$$Z = \begin{bmatrix} f(x_1, y_1) & f(x_2, y_1) & f(x_3, y_1) & f(x_4, y_1) \\ f(x_1, y_2) & f(x_2, y_2) & f(x_3, y_2) & f(x_4, y_2) \\ f(x_1, y_3) & f(x_2, y_3) & f(x_3, y_3) & f(x_4, y_3) \\ f(x_1, y_4) & f(x_2, y_4) & f(x_3, y_4) & f(x_4, y_4) \end{bmatrix}$$

Since \mathbf{z}_1 comes from the 15-th column of Z , and since the 15-th value of \mathbf{x} is -1.2 , the cross-sectional plot we've just seen, properly scaled, is the conditional density of Y given $X = -1.2$. From your textbook, the conditional mean for Y given $X = x$ is computable via the formula

$$E[Y|X = x] = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (x - \mu_X).$$

Plugging in $\mu_Y = \mu_X = 0$, $\rho = 1/2$, $\sigma_X = \sigma_Y = 1$, and $x = -1.2$, one obtains

$$E[Y|X = x] = -0.6.$$

This is about where the plot is centered. The conditional standard deviation is

$$\sigma_{y|x} = \sqrt{1 - \rho^2} \sigma_Y = \sqrt{3}/2.$$

The peak value of the conditional density curve $f_{Y|X}(y|x = -1.2)$ should therefore be

$$\frac{1}{\sqrt{2\pi}\sigma_{y|x}} = 0.4607.$$

So, you would have to scale the cross-sectional plot by a factor of

$$\frac{0.4287}{M},$$

where M is the peak value of the cross-sectional plot (about 0.09).

- The following script evaluates M and the scaling factor that is required:

```
X=-1.2; Y=-0.6; rho=1/2;
M=1/(2*pi*sqrt(1-rho^2))*exp(-(X.^2-2*rho*X.*Y + Y.^2)/(2*(1-rho^2)))
M =

    0.0895
scaling_factor=1/(M*sqrt(2*pi)*sqrt(0.75))
scaling_factor =

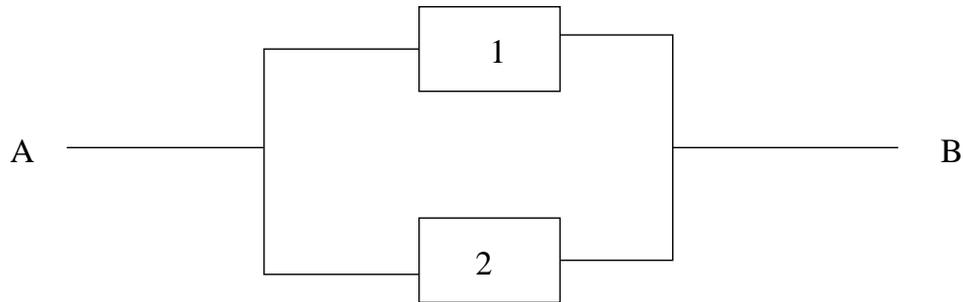
    5.1497
```

So, just scale our earlier curve by a factor of 5.1497. This will give a genuine Gaussian density curve (one for which the area underneath is equal to one). It will be the desired conditional density.

8.4 Exp 4: Expected Lifetime of Relay Circuits

The lifetime of a type 1 relay switch is exponentially distributed and the expected lifetime is 500 hours. The lifetime of a type 2 relay switch is exponentially distributed and the expected lifetime is 300 hours.

Example 9. A type 1 switch and a type 2 switch are placed in parallel to form a relay circuit:



Run the following Matlab script, which estimates the expected lifetime of the relay circuit.

```
t1=-500*log(rand(1,50000));  
t2=-300*log(rand(1,50000));  
t=max([t1;t2]);  
mean(t)
```

Do you understand what each line of code is doing?

Example 10. We show you in this example a really cool way to obtain the exact expected lifetime for the parallel circuit of the preceding example. If the switches are closed with respective probabilities p_1, p_2 , then we know from Chapter 1 that the overall circuit will work with probability

$$1 - (1 - p_1)(1 - p_2)$$

In this expression, plug in

$$p_1 = \exp(-a_1 t), \quad p_2 = \exp(-a_2 t),$$

where

$$a_1 = 1/500, \quad a_2 = 1/300.$$

This gives a function we will call $R(t)$:

$$R(t) = 1 - (1 - e^{-a_1 t})(1 - e^{-a_2 t}).$$

Now we do the following integral

$$\int_0^{\infty} R(t) dt.$$

Perform this integration by running the following Matlab script:

```

clear
a1=1/500;
a2=1/300;
syms t
p1=exp(-a1*t);
p2=exp(-a2*t);
I=int((1-(1-p1)*(1-p2)),0,inf);
double(I)

```

Compare the result with the estimated expected circuit lifetime found from Example 9. Are you surprised?

Example 11. A type 1 switch and a type 2 switch are placed in series to form a relay circuit:



Run the following Matlab script, which estimates the expected lifetime of the relay circuit.

```

t1=-500*log(rand(1,50000));
t2=-300*log(rand(1,50000));
t=min([t1;t2]);
mean(t)

```

Do you understand what each line of code is doing?

Example 12. Let us see if the cool trick of Example 10 will also work for the series circuit above. If the switches are closed with respective probabilities p_1, p_2 , then we know from Chapter 1 that the overall circuit will work with probability

$$p_1 p_2$$

In this expression, plug in

$$p_1 = \exp(-a_1 t), \quad p_2 = \exp(-a_2 t),$$

where

$$a_1 = 1/500, \quad a_2 = 1/300.$$

This gives a function we will call $R(t)$:

$$R(t) = (e^{-a_1 t})(e^{-a_2 t}).$$

Now we do the following integral

$$\int_0^{\infty} R(t) dt.$$

Perform this integration by running the following Matlab script:

```
clear
a1=1/500;
a2=1/300;
syms t
p1=exp(-a1*t);
p2=exp(-a2*t);
I=int(p1*p2,0,inf);double(I)
```

Compare the result just obtained with the estimated lifetime from Example 11. Are you surprised?

Example 13. A relay circuit is formed from 3 switches in parallel. Two of the switches are type 1 and the remaining one is type 2. Using Example 9 as a guide, run a Matlab script to estimate the expected lifetime of the relay circuit. Using Example 10 as a guide, run a Matlab script which will give the EXACT expected lifetime of the relay circuit.

Example 14. A relay circuit is formed from 3 switches in series. Two of the switches are type 2 and the remaining one is type 1. Using Example 11 as a guide, run a Matlab script to estimate the expected lifetime of the relay circuit. Using Example 12 as a guide, run a Matlab script which will give the EXACT expected lifetime of the relay circuit.

EE 3025 S2007 Recitation 8 Lab Form

Name and Student Number of Team Member 1:

Name and Student Number of Team Member 2:

Name and Student Number of Team Member 3:

Study Experiment 4 carefully. I will ask a question concerning estimating the expected life-time of a relay circuit.